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General Theory of the Firm: Business Investment Decision

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by

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Abstract:

To overcome the fragmentary nature of the investment theory in the economic literature, this paper is aimed at developing a more holistic analysis framework of business investment decision. It used deductive reasoning based on the premise that investment decision of the company results from a compromise between managers, lenders and shareholders, with profitability and funding constraints. Thus, by gradually lifting the classical assumptions relating to capital markets, several theoretical results have been obtained, of which four can be considered as major. According to the first result, for each of the individual funding sources, the firm's optimal investment strategy is such as the sum of the marginal capital productivity and the marginal transaction costs of the capital market is equal to the sum of the user cost of capital, the marginal transaction costs of the labor market and the marginal opportunity cost of funding constraints. Considering the equity market and the lending funds market, the second result shows that the optimal investment strategy is such as the total marginal costs of financing minus the total marginal transaction costs on these markets are equal. Thus, the third result reveals that the optimal investment strategy depends on the financing structure if and only if the capital markets are hetero-expensive. According to the fourth result, the optimal funding structure is such as the investor's marginal preference for equity over lending funds is equal to one plus the ratio between, on the one hand, the total marginal financing costs minus the marginal transaction costs of potential financial guarantees; and on the other hand, the marginal transaction costs of the equity-financed investment. The fifth result shows that the relationship between investment and interest rate is not monotonous: it is positive when funding constraints are high and net capital profitability is high enough, otherwise it is negative.

Résumé : Théorie générale de la firme : la décision d'investissement des entreprises

Pour pallier le caractère fragmentaire de la théorie de l'investissement, ce papier s'est fixé pour objectif de développer un cadre d'analyse plus global de la décision d'investissement des entreprises. Basée sur le postulat de la Théorie générale de la firme, la méthode hypothético-déductive a été utilisée en partant de l'idée que la décision d'investissement de l'entreprise résulte d'un compromis entre le manager, les prêteurs et les actionnaires, sous les contraintes de rentabilité et de financement. Ainsi, en levant progressivement les hypothèses classiques relatives aux marchés des capitaux, plusieurs résultats théoriques ont été obtenus dont cinq peuvent être considérés comme majeurs. Le premier résultat indique que pour chaque source de financement, la stratégie optimale d'investissement de l'entreprise est telle que la productivité marginale du capital plus les coûts marginaux de transaction du marché des capitaux soit égale au coût d'usage du capital, plus les coûts marginaux de transaction du marché du travail et le coût marginal d'opportunité des contraintes de financement. En considérant le marché des fonds propres et celui des fonds prêtables, le deuxième résultat montre que la stratégie optimale d'investissement est telle que les coûts marginaux totaux de financement moins les coûts marginaux totaux de transaction sur ces marchés soient égaux. Ainsi, le troisième résultat révèle que la stratégie optimale d'investissement dépend de la structure de financement si et seulement si les marchés des capitaux sont hétéro-dispendieux. Selon le quatrième résultat, la structure optimale de financement est telle que la préférence marginale de l'investisseur pour les fonds propres par rapport aux fonds prêtables soit égale à un (1) plus le ratio entre, d'une part, l'écart des coûts marginaux totaux de financement moins les coûts marginaux de transaction des garanties financières éventuelles et, d'autre part, les coûts marginaux de transaction de l'investissement financé par des fonds propres. Le cinquième résultat montre que la relation entre l'investissement et le taux d'intérêt n'est pas monotone : elle est positive lorsque les contraintes de financement sont fortes et que la profitabilité nette du capital est suffisamment élevée, sinon elle est négative.

Keywords: Firm, investment, transaction costs, funding constraints, financial guaranties.
Mots clés : Firme, investissement, coûts de transaction, contraintes de financement, garanties financières.

JEL classification: D21, E22

Contents

1. Introduction

Since the simple accelerator principle stated by Clark (1917), economists have made significant progress in analyzing the determinants of business investment. In addition to the user cost of capital or the interest rate highlighted by the basic neoclassical model, the profitability of investment (Tobin, 1969), profit (Malinvaud, 1987), financing and the political and institutional environment (Lucas 1993 and Harris 2002) are recognized today as determinants of business investment. The idea of the multiplicity of determinants of corporate investment is all the most accepted as the World Bank, through its Doing Business program, acts in several fields in order to promote private investment in developing countries (the labor market, financial system, fiscal policy, macroeconomic and political framework, regulations and administrative procedures for setting up businesses, etc.).

However, progress made in the analysis of the determinants of investment has not made it possible to obtain a global theoretical framework for the investment behavior of companies. Indeed, investment theory remains fragmented. Consequently, to identify all of the potential determinants of business investment (capital user cost, funding constraints, transaction costs, profit or profitability, etc.), it is necessary to have recourse to several theoretical frameworks. However, the various theories of investment remain difficult to be integrated due to their assumptions and/or their analytical frameworks which are not always convergent.

One of the drawbacks of this situation is that the interest rate continues to be considered as the sole determining factor of corporate investment in macroeconomics, although empirical evidences show the contrary (Naboulet and Raspiller 2006). Also, some authors (Greenwald, Stiglitz and Weiss 1984) argue that imperfections of information in capital market can lead to credit rationing, so that it is the quantity of credit supply, not its cost, which determines the quantity of investment.

Thus, despite the theoretical advances, setting-up a global theoretical framework for business investment behavior remains a challenge. Faced with this challenge, the purpose of the article is to propose a general framework for corporate investment behavior, which is positioned as the logical continuation of previous developments in the general theory of the firm (Zerbo 2016).

As a reminder, the general theory of the firm refutes the profit maximization hypothesis and postulates that firms operate on the compromises basis between its stakeholders in the different markets. Thus, according to Zerbo (2016, 2018a and 2018b), the demand for labor is determined on the compromise basis between the employer and the workers under constraint of production capacities; it depends in the short run on gross profit. In the medium and long run, demand for labor as well as gross profit depends on real wages and capital stock. Then, the gross profit function resulting from the compromise on the labor market is internalized in the investment decision process which, itself, is the result of compromise between the stakeholders of the company in the capital market, namely managers, shareholders and lenders.

Relying therefore on the postulate of the General Theory of the Firm (GTF), this research aims at developing a more holistic theoretical framework of the investment decision of companies, taking into account both the user cost of capital, transaction costs linked to market imperfections, funding constraints, collaterals, taxation and the diversity of funding sources. To limit the complexity of the process and for pedagogical reasons, this paper is limited to one period. The time dimension of the investment decision will be included in a future paper.

This paper is structured in six sections. Before setting out the new theoretical framework for analysis of business investment decision (section 2), a brief literature review is done (section 1) and after corporate investment decision according to the new theoretical framework is analyzed (section 3). In three last sections, three main relationships are examined (i) between corporate investment behavior and the financing structure, (ii) between investment and interest rate according to the level of funding constraints, and (iii) between investment and retained earnings.

2. Literature review on corporate investment behavior

In the economic literature, several theoretical models have been developed to explain the investment behavior of companies. The main models are: (i) the accelerator theories, (ii) the neoclassical profit maximization model, (iii) Tobin's Q theory, (iv) the financial approaches of investment decision and (v) transaction cost theory.

1. *The Accelerator principle*

Rigorously developed for the first time by Clark in 1917, the accelerator principle considers that the capital stock is linked to expected production by a proportionality relationship as indicated in the relation 1 where K_t, Y_t and v denote respectively the capital stock of period t, the total output at t and the capital ratio.

$$K_t = vY_t \quad (1)$$

$$K_t = I_t + (1-\delta)K_{t-1} \quad (2)$$

Knowing that the capital stock at period t is defined by the relation 2 where the investment in period t (I_t) is given by the relation 3 which indicates that the quantity of the investment at t is equal to the expected variation of total output multiplied by the capital ratio plus the quantity of capital depreciation in period t-1. It's the simple accelerator principle that net investment of companies is proportional to the expected change in demand for goods and services.

$$I_t = v\Delta Y_t + \delta K_{t-1} \quad (3)$$

From relations 1 and 3, it can be deduced that the rate of capital accumulation is equal to the rate of expected economic growth (g^a), plus the depreciation rate of capital (relation 4).

$$\frac{I_t}{K_{t-1}} = g^a + \delta \quad (4)$$

Thus, the principle of the simple accelerator explains the investment behavior of companies by their needs (i) to renew depreciated capital and (ii) to increase production capacities to meet the increase in demand for goods and services. However, the capital ratio is exogenous; anything that reduces the explanatory power of the model because the investment behavior depends in particular on the different investment costs.

Thus, following Clark (1917), Grossman (1929) upgraded the simple accelerator model by "endogenizing" the capital ratio v. He starts from the assumption that the enterprise maximizes its production function under the constraint of production cost and deduces that the capital ratio is a function of the relative cost of labor compared to capital, i.e. w/r, where w and r denote respectively the rate of wage and the interest rate (relation 5).

$$I_t = H\left(\frac{w}{r}\right)\Delta Y_t + \delta K_{t-1} \quad (5)$$

This development made by Grossman (1929) did not fundamentally change the principle of the simple accelerator. Indeed, in the model, the adjustment of the real capital stock to the desired stock remains instantaneous. However, in reality, companies prefer to slowly adjust their real capital stock to the desired stock because, on the one hand, any act of investment generates irreversible transaction costs

which constitute an immediate loss of production for the company. These transaction costs are linked in particular to the analysis of the investment project, the approval and programming of the project, the search for financing (internal and/or external), the installation of equipment, the adaptation of staff (qualitative and quantitative). On the other hand, companies do not have all the necessary capacities to forecast with certainty the variations of demand for goods and services in the medium and long run; so they make their investment operations with prudence.

In view of this weakness of the simple accelerator model, Koyck (1954) developed the flexible accelerator model which consists in relating investment, not to the simple current variation in income, but to a weighted average of past developments. He obtains the relation 6 where the parameter λ, between 0 and 1, represents the speed of adjustment of the real stock to the desired stock of capital. The introduction of this parameter by Koyck attenuated the speed of adjustment of the model by reducing the coefficient of acceleration and by introducing inertia into the dynamics of investment.

$$I_t = \lambda v \Delta Y_t + (1-\lambda) I_{t-1} \qquad (6)$$

Empirical studies (Michaudon & al. 1998) are unanimous on the role of expected output on the evolution of investment. However, it is recognized that the accelerator principle explains the simple "mechanical relationship" that exists between investment quantity and the level of production: to produce more, one must invest more. It cannot allow to identify or to analyze the corporate investment behavior in an environment characterized by many changing and indeterminate factors such as opportunities, risks and investment alternatives, user cost of capital, transaction costs, financial constraints and the degree of liquidity or reversibility of the investment. Thus, different theoretical approaches have been developed on the investment behavior of companies: orthodox approaches (Jorgenson 1963 and Tobin 1968), financial approaches (Modigliani-Miller 1958) and the transaction cost approach.

2. *The neoclassical model of investment*

Initiated by Jorgenson (1963), the neoclassical model of investment is based on the behavior of maximizing the profit of companies without any constraint. It is assumed that companies, operating in competitive markets, produce goods and services according to a production technology which depends on labor (L) and capital (K), in order to generate profits after having paid these inputs. Relations 7 and 8 give respectively the production and profit functions, where p denotes the price level of goods and services, w denotes the wage rate, δ is the capital depreciation rate and r is the interest rate.

$$Y = F(K, L) \qquad (7)$$

$$\pi = pF(K, L) - wL - (\delta + r)K \qquad (8)$$

The first order conditions of profit maximization give the system of equations 9 which indicates that profit is maximum when the marginal productivity of each input is equal to its real cost. Two functions of investment demand can be identified depending on whether one is in the short run or in the long run.

$$\begin{cases} \dfrac{\partial F}{\partial L} = \dfrac{w}{p} \\ \\ \dfrac{\partial F}{\partial K} = \dfrac{(\delta + r)}{p} \end{cases} \qquad (9)$$

From these first-order conditions of gross profit maximization, it can be deduced that at the optimum, the marginal rate of technical substitution of labor for capital ($MRTS_{LK}$) is equal to the relative

cost of labor compared to capital (relation 10). Thus, in the short run, when the level of production is constrained by markets, the demands for inputs are determined by their relative costs.

$$MRTS_{LK}(K, L) = \frac{w}{(\delta + r)} \quad (10)$$

Thus, under the assumptions of the concavity of production technology, the demand for investment decreases in the short-run with the relative cost of capital (relation 11). According to this model, when the relative cost of capital decreases, employers would be willing in the short run to reduce the quantity of labor to invest more; while when it increases, they would reduce their investments in favor of the labor.

$$K_{SR}^{d} = K_{SR}\left(\frac{(\delta + r)}{w}\right) \quad (11)$$

In the long run where the level of production is no longer constrained by markets, the demand for each input would be a function of its real cost (system of equations 9). Thus, in the long run, the demand for investment would decrease with the real user cost of capital (relation 12).

$$K_{LR}^{d} = K_{LR}\left(\frac{\delta + r}{p}, \frac{w}{p}\right) \quad (12)$$

This basic neoclassical model is criticized for its assumptions that are too far disconnected from the reality of business. Also, a substitution between capital and labor in short run according to the change in their relative costs is difficult to conceive in practice given the long times that the companies need to operate their investment and the irreversible nature of corporate investment. Furthermore, empirical studies fail to highlight the negative theoretical relationship between the user cost of capital and investment. On the contrary, some empirical studies have led to a positive relationship between the short-run interest rate and investment (Gennaioli et al. 2015, Sharpe and Suarez 2014).

3. *Tobin's Q theory*

Unlike the two previous theories in which expectations do not play a large role, Brainard and Tobin (1968), and Tobin (1969) have developed a theoretical model which links investment to the ratio between the market value of the company and the replacement value of its capital stock. According to this theory, the more the market value of the company is higher than the replacement value of its capital stock, the more it is profitable for the company to invest because an additional unit of investment creates a financial value which is greater.

$$Q = \frac{M}{K} \quad (13)$$

Thus, for the choice of investments, Tobin (1969) proposes to follow the ratio, called Q-average of Tobin, given by the relation (13) where M is the market value of the company at period t and K is the value of capital replacement. When this ratio is higher than one, the stock market anticipates a profitability of the investment beyond its cost, so the investment would be profitable. Otherwise, the investment would not be profitable.

Tobin's Q theory is equivalent to the basic neoclassical theory incorporating an investment adjustment cost function and the constraint of capital stock renewal. Indeed, let A and *s* be respectively

the marginal cost of capital adjustment and the marginal cost of capital replacement[1]. The company's program consists in maximizing profit under constraint (relation 14).

$$\begin{cases} \underset{I}{Max}\left[F(K,L)-\frac{w}{p}L-rK-AsI\right] \\ s/c \quad \delta K \le I \end{cases} \qquad (14)$$

Knowing that *s* is the real cost of capital replacement, the capital stock K at t is equal to the initial capital stock K_0 plus the quantity of the investment made at t, i.e. *s*I. Thus, the Hamiltonian of the optimization program 14 is given by the relation 15, where λ is the Hamiltonian coefficient. Here this coefficient represents the opportunity cost of capital borne by the company.

$$H(I)=F(K_0+sI,L)-r(K_0+sI)-AsI-\frac{w}{p}L-\lambda\left(\delta K-I\right) \qquad (15)$$

The first order conditions of the program give the relation 16, which indicates that the ratio between the opportunity cost of capital and its marginal replacement cost, namely Tobin's q-marginal (q_m), is an increasing function of investment I.[2]

$$q_m=\frac{\lambda}{s}=r+A-\frac{\partial F}{\partial K}(I) \qquad (16)$$

So, it is deduced that the investment is increasing with the q-marginal of Tobin as illustrated in relation 17

.

$$I=I(q_m) \quad avec \quad \frac{\partial I}{\partial q_m}\ge 0 \qquad (17)$$

Thus, for this theory of Tobin's Q, the profitability of capital is the main determinant of corporate investment. However, it has an important weakness linked to the efficiency of the stock market which means that the market value of a company is equal to the exact value of this company. It attributes to the stock market the ability to be able to exactly assess the financial value of each company at any moment; which assumes that the information is perfect and the capacities of stakeholders to make optimal decisions in this market are endless. Also, because of the speculative phenomena in the stock market which cause a high volatility of the value of companies, the approach of Tobin's Q does not give a satisfactory explanation of changes in the investment. Schaller (1990) observed through empirical results that Tobin's Q does not explain the evolution of investment in the United States and the United Kingdom.

4. *Financial approaches of corporate investment*

Modigliani-Miller's theorems (1958) actually mark the beginning of financial approaches of business investment. Comparing two identical companies which differ simply by the composition of their resources (equity and borrowing), Modigliani and Miller show that under the assumption that the markets are perfect and without taxes, the value of a company is independent of its financial structure. This implies that under perfect market conditions, investment decisions are not influenced by the company's financial decision.

[1] To simplify the presentation, let us suppose that A and s are constant.

[2] Under the assumption of concavity of the production function, the marginal productivity of capital decreases with investment, therefore its opposite is increasing with investment.

Although it was a fundamental theoretical result of corporate finance, the Modigliani-Miller theorem has been criticized for the lack of realism in its assumptions. Markets are not perfect in reality; so the question is how the investment decision of a company depends on its financial policy. Thus, following Modigliani and Miller (1958), several authors argue that the financial conditions of the company constitute determinants of investment (Stiglitz and Weiss 1981, Bernanke and Blinder 1988, Bernanke, Gertier and Gilchrist 1999).

According to Bernanke, Gertier and Gilchrist (1999), when companies are highly dependent on external financing, the cost of funding is higher; so, the investment is held back. The effect of financial variables on investment was highlighted for the United Kingdom by Hall (2001), for the United Kingdom, France and Germany by Ashworth and Davis (2001). Thus, too many debts in terms of equity and too low profitability are among the blocking factors of investment.

According to Stiglitz and Weiss (1981), credit rationing, favored by imperfections in the credit market, has a negative impact on corporate investment. Critics say that if the only asymmetric information between the lender and the borrower relates to the risk of the project, then the use of collateral solves the problem of credit rationing. On the contrary, Kiyotaki and Moore (1997) have shown the existence of credit cycles when certain companies are constrained in the credit market and have to use their capital stock as collateral. Indeed, in the event of a negative productivity shock, the value of the capital stock of the affected companies decreases; if they can only borrow by guaranteeing their productive assets, then the decrease in the value of these collaterals leads to a decrease in investment.

Empirical tests by Kashyap, Stein and Wilcox (1993) based on a model close to the model of Bernanke and Blinder (1988) show that a change in monetary policy has an effect on investment that is quite distinct from interest rate effect. Empirical work by Naboulet and Raspiller (2006) invalidates the expected relationship between interest rate and the investment decision. Also, several empirical studies (Kashyap, Lamont and Stein 1994, Carpenter, Fazzari and Petersen 1994) have highlighted the role of cash flow and/or profits as an explanatory factor for companies' investment behavior.

5. *Transaction cost theory and business investment decision*

The neoclassical investment model assumes that economic agents are rational in the absolute sense, markets are efficient and physical assets are not specific, so they can be redeployed as desired. Therefore, the transactions necessary for the act of investing in the company entail no cost. Conversely, for Coase (1937) and Williamson (1975), the stakeholders of the company establish contractual transactions because their rationality is limited, opportunism is possible and the physical assets are specific. So, to make a transaction, the company has to incur costs, called transaction costs.

According to Simon (1961), the cognitive capacities of individuals are limited. In addition, the human judgments are fallible, in particular, because of the limited effort and time that an individual can devote to a decision (Sah 1991). This is how Williamson (1975) argued that the main problem for economic organizations is to face and deal with uncertainty, and that this is all the more crucial as the environment is changing. Then, based on Simon (1961), Williamson (1975) recalls the principle of limited rationality according to which the economic agents make choices intentionally rational, but inevitably limited in the ex-ante analysis of the consequences of decisions because of the limits in human capacities to process information. Thus, according to Simon (1961), because of their limited capacities to forecast the different states of their environment over an unlimited time horizon, the economic agents favor short-run investment.

For Marris (1964), Williamson (1964), Jensen and Meckling (1976), the utility function of economic agents does not depend only on perfect pecuniary substitutes; it also depends on the advantages associated with prestige, discretionary latitude, empire building or self-realization which are

not taken into account in substantive rationality. As a result, the choice of investments is no longer made exclusively on the basis of discounted cash-flows, but also from these other elements which are an integral part of the company's objectives. Similarly, North (1990) argues that the institutional environment of the company is made up of game rules, including legal ones, which constrain investment choices. Moreover, according to Charreaux (2000), beyond the objective characters of the environment, the cognitive and subjective representation that each stakeholder has of the company also conditions the investment process.

In addition to limited rationality, opportunism is possible in trade if it is recognized that each stakeholder is autonomous. Beyond its natural character favored by the differentiated positions of the stakeholders in relation to the company, the asymmetric information can be maintained and/or reinforced by one of the stakeholders for a strategic purpose before the conclusion of the transaction (adverse selection) and/or after the conclusion of the transaction (moral hazard). For example, being more informed about the risks of his investment project, the manager has no interest in disclosing them entirely to his lenders and would be ready to borrow at a high rate when the risk is high. Being relatively less informed about the risk of the project, the banker will tend to finance bad projects at the expense of good projects (adverse selection). So, in the presence of adverse selection, the market is not efficient (Akerlof 1970).

In general, company managers have more information about their investment plans than company shareholders and lenders; which promotes opportunistic behavior and generates agency costs. Therefore, according to positive accounting theory, the main function of financial reporting is to compel managers to act in the interest of shareholders (Watts and Zimmermann 1978). Indeed, according to Ball and Shivakumar (2005), one of the inherent characteristics of financial reporting is to help boards of directors to have better control over managers' decisions by promptly detecting opportunistic behavior on the part of the latter, particularly with regard to investment. Also, the moral hazard faced by shareholders and lenders explains the existence of internal control and regular audits, especially in large companies. The implementation of these essential control instruments is unfortunately expensive, so it generates significant transaction costs.

Other transaction costs are linked to the fact that physical assets are specific. This specificity of physical assets makes the investment decision irreversible (Malinvaud 1987). Indeed, the act of investing involves not only the acquisition of physical assets in the market, but also the transformation of this indeterminate physical assets into specific physical assets, specific to the use of each company. The specification of physical assets may in particular require the adaptation of equipment, the training of personnel, as well as irreversible modifications of the physical environment. Also, the inefficiency of the second-hand markets is a source of irreversibility of the investment decision given the large difference between the purchase price and the resale price of the physical assets. So it is expensive to invest as well as to disinvest because of the specificity of the physical assets. Given the uncertainty that characterizes future trends in demand for goods and services, the irreversibility of the investment decision increases companies prudence and, therefore, slow adjustments to the capital stock.

Different theories on transaction costs have made economists aware of the limits of neoclassical theory in general and neoclassical models of investment in particular. However, transaction cost theories are criticized for their lack of clarity (Fischer 1977) and for ambiguity of transaction costs assessment (Slater and Spencer 2000, Klein 2004); anything that has not yet allowed transaction costs to be taken into account in macroeconomic analysis and in practice.

This brief literature review shows that the investment theory has grown considerably after the basic neoclassical model. In addition to the user cost of capital, several other determinants of corporate investment behavior have been highlighted in the various investment theories. These include expected demand, profitability, financial conditions, access to finance and transaction costs. Despite this advance, investment theory remains fragmented, with theoretical contributions whose hypotheses and/or results seem to contradict each other. Thus, to each determinant of investment mentioned above corresponds a specific theory which often diverges from the other theories.

3. New Framework for analyzing business investment decision

The objective of this section is to develop a theoretical framework for the investment behavior of companies which is positioned as a continuation of previous reflections of the general theory of the firm (Zerbo 2016). Indeed, the first developments focusing on the behavior of labor demand (Zerbo 2016, 2018a, 2018b), the general theory of the firm would be unfinished if it is not extended to the corporate investment behavior.

The basic idea of the General Theory of the Firm is that in the markets, participants make deals or negotiate compromises between themselves: they negotiate, sign contracts, agreements or conventions, and execute them in an environment characterized, among other things, by imperfect markets, asymmetric information and the existence of privileged relationships. So, for companies, their labor demand behavior is strong-minded by the labor market compromise process, while their investment decision is determined by the capital market compromise process.

Thus, this section contains three subsections. The first subsection recalls the basic premise of the general theory of the firm. The second subsection presents the primary trade-off process for determining the demand for labor. The third subsection presents the capital compromise model which leads to the investment decision of companies.

1. *The basic postulate of the General Theory of the Firm (GTF)*

The general theory of the firm is based on the idea that the company is an entity, composed of the employer or managers, employees and possibly shareholders, which owns assets, contracts, develops and manages specific know-how, promotes compromise between stakeholders, produces goods and/or services to generate income which is distributed. Also, the company may maintain privileged relationships with suppliers and especially with banking or financial institutions particularly for the management of its treasury and the financing of its investment projects.

Contrary to the neoclassical concept, the company does not only serve the interests of the employer or owners, namely the maximization of profit. It aims to satisfy all stakeholders so that it performs sustainably in its value creation function. Although conflicting, interests of stakeholders are interdependent. Indeed, as much as the employer wishes to make more profit, it is in his interest (i) that the employees are relatively satisfied so that the labor productivity would be high and (ii) that the shareholders and the lenders are also satisfied so that they would continue to support the company's investment projects. Conversely, in order to keep their jobs and have high wages, employees have an interest that the employer, the shareholders and the lenders are satisfied so that they would continue to support the company and, thus, to guarantee decent work. Likewise, as much as the shareholders want to have high dividends, they also have an interest in ensuring (i) that the company has the capacity to invest again, (ii) that the managers and employees are satisfied with their working conditions in order to improve sustainably the financial results, and (iii) that the lenders are suitably paid so that they would continue to support the company's investment projects. As for the lenders of the company, as much as they want to get high interest rates for their loans, as much they have an interest so that the company can be durably efficient in order that it can honor its commitments over time.

As a result, the interests of the company's stakeholders appear a priori contradictory, but they are interdependent. This situation forces the company to operate on the compromises basis between the stakeholders (negotiations, agreements/contracts, conventions). Operating on compromises basis does not exclude the strategic or opportunistic behaviors by stakeholders. The asymmetric information and the disparity of bargaining power between the stakeholders encourage such behaviors, not only during negotiations, but also during the execution of the compromise. Also, because of the changes that can

occur in the relationships between stakeholders, especially on the informational and institutional levels, the compromise is not static; it is dynamic. For example, the updating of information on the earnings or on the opportunistic behavior of a stakeholder can lead to calling into question the state of compromise in force by the other stakeholders and, thus, cause new negotiations to establish a new state of compromise.

Thus, the state of compromise of the company at a given time depends not only on the institutional, legal and information environment, but also on the economic and social environment in which the company operates. For example, the position of each stakeholder in the negotiations will depend in particular on the fact that the economic environment provides him/her more or less other alternatives to achieve their goals (whether he has other choices or not). Also, the degree of confidence between the stakeholders and the degree of rationality or altruism of the stakeholders, as well as the social relationships between them influence the state of compromise.

In view of these elements, the general theory of the firm considers that each company is characterized by an implicit function of compromise, i.e. "a subjective utility function", which it seeks to optimize so that each stakeholder feels satisfied. This is in line with Williamson's (1975) principle of limited rationality, according to which economic agents make choices that are intentionally rational, but inevitably limited, particularly because of the limits in their capacities to access and process information, as well as the limits imposed by the institutional, legal, relational and social environment. Obviously, just like a state of compromise, the function of compromise is characterized by the state of the institutional and legal environment, economic and social environment and informational environment, in which the company operates. Therefore, the function of compromise changes structurally with these elements. The measurable objectives of the stakeholders constitute the arguments of this compromise function, among others, retained earnings, wage rate, employment, investment, shareholders return rate, collaterals and loaners interest rate.

In the operating of the company, two interdependent stages of negotiation or compromise can be distinguished. On the one hand, there is the "primary compromise" which relates to the distribution of added value between the wage bill and the gross operating surplus. On the other hand, there is the "capital compromise" which relates to the distribution of the corporate income between interest paid to lenders, dividends and retained earnings[3].

Previous developments in the general theory of the firm (Zerbo, 2016, 2018a, 2018b) were devoted to the primary compromise, which allowed, among others, to identify the demand for labor. This paper addresses the capital compromise. Insofar as the two stages of compromise are linked, a reminder of the primary compromise is essential.

2. *The process of primary compromise in the firm*

According to Zerbo (2016), the compromise between employers and workers (primary compromise) mainly relates to (i) the average level of real wages (w/p), (ii) the level of employment (L) and (iii) real gross profit (π), under the constraint of production possibilities. Thus, given the institutional, informational, economic and social environment, the stakeholders seek to reach the optimal compromise, constrained by production possibilities.

Let U be the function of primary compromise given by relation 18 and F the production function of companies given by relation 19. Then, the primary compromise program of enterprises is given by relation 20.

[3] Retained earnings ("autofinancement" in French) is the accountant balance that denotes the self-financing possibilities of a company. The retained earnings are equal to the gross operating surplus minus financial expenses, minus corporate income tax and minus distributed dividends.

$$U = U(\pi, L, w/p) \tag{18}$$

$$Y = F(K, L) \tag{19}$$

$$\begin{cases} Max\, U(\pi, L, w/p) \\ u/c \quad \pi + (w/p)L - F(K, L) \leq 0 \end{cases} \tag{20}$$

This primary compromise program determines, on the one hand, the process of wage negotiation and, on the other hand, the behavior of labor demand of the company. In fact, wage negotiations precede the demand for labor by companies; that is, the stakeholders (employers and employees) agree on the remuneration of the workforce before it is used. Then, the corporate program can be distinguished in two stages. The first stage concerns wage negotiations which determine the real wage level and the second stage relates to the determination of labor demand.

Given the institutional, regulatory and informational environment of the labor market characterized by the primary compromise function U, as well as the quantity of labor necessary to produce a unit of production quantity (L/Y), wage negotiations relate to real wages (w/p) and the share of profit per production unit (π/Y)[4], under the constraint of the wealth created. Indeed, to fix wages, the labor productivity (conversely the labor per production) is considered by the stakeholders as a given, even if it is imperfectly known and it is subject of moral hazard. Employers wish to remunerate this given labor productivity at a real wage level which would guarantee them an acceptable level of work effort and a high gross profit ratio (π/Y), while employees aim a relatively high real wage for this level of work effort they must provide. As their interests are a priori contradictory and interdependent, employers and workers must negotiate to fix the level of real wages.

Thus, from the primary compromise program of companies (relation 20), it is deduced the process of wage negotiation given by relation (21).

$$\begin{cases} \underset{\pi/Y,\, w/p}{Max}\; U\left(\frac{\pi}{Y}, \frac{w}{p}, \frac{L}{Y}\right) \\ u/c \quad \frac{\pi}{Y} + (w/p)\frac{L}{Y} \leq 1 \end{cases} \tag{21}$$

The first-order conditions of this optimization program give the system of equations (22), which indicates that the real compromise wage is such as the marginal rate of substitution of real wage for profit per production unit ($MRS_{w\pi}$) is equal to the quantity of labor per production unit, i.e. the slope of the constraint line in the plane (w/p; π/Y). The compromise point ((w/p)*; (π/Y)*), solution of the system of equations (22), is such as the desire to earn an extra penny on real gross profit per unit of production is equal to the desire of an employee to earn an extra penny on real wages.[5]

$$\begin{cases} MRS_{w\pi}\left(\frac{\pi}{Y}, \frac{w}{p}, \frac{L}{Y}\right) = \frac{L}{Y} \\ \frac{\pi}{Y} + (w/p)\frac{L}{Y} = 1 \end{cases} \tag{22}$$

Furthermore, the system (22) allows to determine the compromise point of salary negotiation as being the point of intersection of the compromise curve of salary negotiation (first equation) and the constraint line of income (second equation) in the plane (w/p; π/Y). Doing the total differential of the

[4] The ratio (π/Y) is a measure of the average gross profitability of companies' activities in the short run.
[5] See Zerbo (2016) for the details on the first order conditions of primary compromise.

first equation of the system (22), it appears that the compromise curve is increasing in the plane (w/p, π/Y).

The resolution of first-order conditions (system of equations (22)) makes it possible to obtain the expression of real wages as a function of labor productivity (Y/L). Replacing, in the first equation of system (22), the ratio of gross profit by its expression given by the constraint of income distribution and differentiating this first equation, it appears that real wages are increasing with labor productivity. Thus, the relation (23) can be written; it is expresses the real compromise wage as a function of labor productivity (Y/L).

$$(w/p)^* = w_r\left(\frac{Y}{L}\right) \tag{23}$$

Once the real wage is fixed through the wage negotiation process, the company maximizes the compromise function in relation to the levels of labor demand and real gross profit, under the constraint of the wealth created (program 24). In fact, for a fixed real wage level (w/p), employers aim for a high real gross profit level by minimizing, as much as possible, the total cost of labor force; while employees aim for a high level of employment which would avoid lay-offs and, at best, reduce the workload per worker. Their interests being a priori contradictory and interdependent, the stakeholders must enter into negotiations to determine, on the basis of the compromise, the level of employment and, thus, the real gross profit.

$$\begin{cases} \underset{\pi,\, L}{Max}\, U(\pi,\, L,\, w/p) \\ s/c \quad \pi + (w/p)L - F(K,\, L) \leq 0 \end{cases} \tag{24}$$

The resolution of this maximization program of the compromise function under the production constraint gives the equations of the system 25.[6] Thus, given the imperfection of information, asymmetric information, the respective bargaining power of stakeholders, labor law and social relationships, the optimal compromise (π*; L*) is solution of the system of equations 25.

$$\begin{cases} MRS_{L\pi} + \dfrac{\partial F}{\partial L} = w/p \\ \pi + (w/p)L = F(K,\, L) \end{cases} \tag{25}$$

The first equation of system (25) gives the conventional function of the labor demand of the company, while the second equation expresses the income constraint. So, under the assumptions of the convexity of the set of compromise possibilities and the concavity of the production function, the demand for labor increases with real gross profit.[7] Furthermore, the econometric tests carried out by Zerbo (2018a) on the data of OECD countries confirm that the demand for labor is increasing with gross profit.

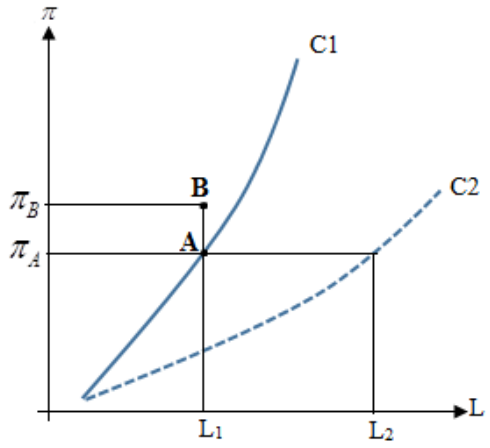
In the system of equations 25, the value of the marginal rate of substitution of labor for gross profit ($MRS_{L\pi}$) at a point (L; π) represents, on the one hand, the marginal cost due to the labor market imperfections supported by employers for the level of real gross profit π and the level of labor demand L? To say simply, $MRS_{L\pi}$ at a point (L, π) represents the marginal transaction cost of the labor market at the levels L and π. At any point on the conventional labor demand curve (C1 in Graph 1), transaction costs are equal to the marginal loss (shortfall) due to imperfections in the labor market (w/p – ∂F/∂L)

[6] The compromise function is concave (or all the possible choice is convex) and the inequality constraints are convex; for the maximization program resolution, Kuhn – Tucker Theorem can be used.

[7] Take the total differential of the first equation of the system 25 by maintaining capital stock and real wage constant.

and, thus, the real gross profit is equal to the effective gross profit. On the other hand, at any point above the labor demand curve, for example at point B, the marginal transaction cost is greater than the marginal loss due to imperfections [8]. As a result, the additional transaction cost that companies would bear to maintain themselves at point B is greater than the shortfall (π_B-π_A) due to stay at point A.[9] Ultimately, the effective gross profit[10] of companies at point B is less than the real gross profit π_B minus the shortfall (π_B - π_A), that is to say that the effective gross profit at point B is less than the gross profit at A, i.e. π_A.

Graph 1: Labor demand curve according to the degree of flexibility of the labor market



Source: Zerbo (2018b)

On the other hand, the $MRS_{L\pi}$ reflects the degree of flexibility of the labor market. The more flexible the labor market becomes, the more the $MRS_{L\pi}$ tends towards 0 and, thus, the system of equations (25) tends towards neoclassical conditions (profit maximization) and the demand for labor of companies becomes less sensitive to variations in real gross profit. Conversely, the more rigid the labor market becomes, the $MRS_{L\pi}$ increases and, thus, the labor demand of companies becomes more sensitive to changes in profit. Then, in plane (L, π), the slope of the labor demand curve will be steep for a relatively more flexible labor market, as illustrated by curve C1 in graph 1; while it will be weak for a relatively less flexible labor market, as illustrated by curve C2.

Solving the system of equations (25) gives the level of labor demand L* of compromise state, which is a function of the real gross profit (π*) and the capital stock (K), as shown in relation (26).

$$L^* = L(\pi^*, K) \qquad (26)$$

Real gross profit is therefore the determining factor of labor demand in short-run. Under the assumptions of the convexity of the set of compromise possibilities and the concavity of the production technology, it is shown that the demand for labor increases with real gross profit.[11]

[8] The $MRS_{L\pi}$ being an increasing function with π and decreasing function with L, the MRS at B is higher than the MRS at A. The level of labor demand being identical at A and B, the marginal loss is identical and equal to (w/p – ∂F/∂L (L1)). As the MRS at A is equal to (w/p – ∂F/∂L (L1)), then the MRS at B (marginal transaction cost at B) is greater than (w/p – ∂F/∂L) (L1), i.e. the marginal loss at B.

[9] At any point of segment]AB] in the graph 1, so that MRS > C_m, with C_m = w/p – ∂F/∂L (L1); as result: $\int_{\pi_A}^{\pi_B} MRS_{L\pi} d\pi > \int_{\pi_A}^{\pi_B} C_m d\pi$

[10] The effective gross profit of companies is equal to the real gross profit minus the total costs due to labor market imperfections (cost of tensions management/labor conflicts, community relations, information, etc.). i.e. the total of transaction costs

[11] To calculate the total differential of the obtained relation in the previous footnote where w/p is constant.

3. *The capital compromise process of companies: the one period model*

Let consider the company has an initial net capital stock K_0, of which the share $\boldsymbol{\alpha}_0$ has been financed by borrowing at an interest rate equal to r_{D0}. The company seeks to seize the new opportunities that arise to increase its production level and generate more income; so, it needs to invest to increase the capital stock. How much investment would be needed to increase its income? Answering this questions raises several others relating to the capital user cost, the transaction costs, the possible cost of extra labor due to the investment, the profitability of the investment, the financing strategy of investment, etc.

To invest, the company can use equity and/or loans. Thus, the company's investment decision involves three main types of economic players: the managers, the shareholders and the lenders. This decision is the result of a compromise between these three economic players about the distribution of the incomes generated by the investment project and the amounts of the investment per financing source.

In this process of negotiating the capital compromise relating to the investment project, (i) the managers want to make more retained earnings, (ii) the shareholders want greater return on their shares and (iii) lenders wish to lend with high interest rates and important collaterals from the managers in relation to their commitments.

So there is an implicit compromise function, which is designated as the "capital compromise function" in this paper. The arguments of the capital compromise function are: the expected retained earnings (G), the net return rate expected by the shareholders (r_{nE}), the net interest rate charged by the lenders (r_{nD}), the respective quantity of the investment financed by equity (I_E) and by debt (I_D), the amount of collaterals required by lenders (B). Noted V, the capital compromise function is given by the relation (27).

$$V = V(G\,;\, r_{nE}\,;\, I_E\,;\, r_{nD}\,;\, I_D\,;\, B) \qquad (27)$$

Thus, this capital compromise function takes into account (i) the purpose of the managers, specifically the quantity of investment ($I_E + I_D$) and the expected retained earnings (G), (ii) the objective of the lenders which is to lend to the company an amount I_D at a net interest rate r_{nD}, with collaterals amount B, as well as (iii) the objective of the shareholders, which is to invest an amount I_F of theirs funds at net return rate r_{nF}. Some properties of the compromise function are given in Box 1.

Box 1: Some properties of the capital compromise function (V)

i) The function V gives a measure of the level of collective satisfaction of the investment stakeholders;

ii) The function V is concave; in other words, the set of compromise possibilities is convex; which means that negotiation allows stakeholders to converge towards intermediate positions;

iii) The partial derivative of V with respect to the retained earnings G measures the marginal satisfaction of the stakeholders when the retained earnings increases at the margin. The more the retained earnings G is characterized by high uncertainties, the more the marginal satisfaction of retained earnings G is low.

iv) The marginal rate of substitution of investment for retained earnings (MRS_{IG}) measures the marginal financial sacrifice made by the company on the retained earnings due to imperfections for an extra unit of investment. Thus, the MRS_{IG} measures the marginal transaction costs of the investment.

For the managers, the objective is to increase the production capacity (the stock of physical capital) in order to generate a high retained earnings, which gives them more possibilities of internal financing of investment in the future. Let δ be the depreciation rate of the capital, τ the rate of corporate income tax and $\boldsymbol{\alpha}_0$ the share of debt in the net capital stock K_0. The retained earnings generated by the company is given by the relation (28). The relation (29) recalls the expression of the gross profit which intervenes in the expression of the retained earnings. The relation (30) holds that the capital stock is equal to the existing net capital stock plus the total of investment.

$$G(I_E, I_D) = (1-\tau)\left[\pi(K_0 + I_D + I_E) - r_D(\alpha_0 K_0 + I_D) - (K_0 + I_D + I_E)\delta\right] - r_F((1-\alpha_0)K_0 + I_E) \quad (28)$$

$$\pi(K) = F(K, L) - (w/p)L \quad (29)$$

$$K = K_0 + I_D + I_E \quad (30)$$

As for lenders, they have a financing supply function according to the category of project (relation 31). The arguments of this financing supply function are, among other things, the net interest rate and the collaterals they would require from the company. The more the managers are able to provide the necessary collaterals and/or pay lenders at a high interest rate, the more willing lenders are to finance their projects.

$$S_{Debt} = \varphi(r_{nD}; B) \quad (31)$$

The amount of collaterals supplied by the managers to the lenders depends on the amount of the investment credit, which is equal to the amount of the investment I_D (relation 32). The supply of collaterals increases with the amount of investment I_D financed with loans.

$$S_{Col} = B(I_D) \quad (32)$$

As regards shareholders, their financing supply depends on the net return they can expect from the investment (relation 33). Thus, the higher the net return rate of the shares, the more the shareholders are ready to provide funds for the investment project.

$$S_{Eq} = \psi(r_{nE}) \quad (33)$$

So, on the basis of these elements, the negotiation process leading to a compromise between the three categories of economic players (managers, lenders and shareholders) around the investment project consists in optimizing the function of capital compromise (relation 27) under the following constraints:

(i) the retained earnings expected by managers is lower than or equal to the retained earnings generated by the company;

(ii) the quantity of investment financed by debt is less than or equal to the loan supply;

(iii) the quantity of equity-financed investment is less than or equal to the equity supply;

(iv) the amount of collaterals obtained by lenders is less than or equal to the company's collaterals supply.

Thus, the general program of capital compromise at a period is given by relation 34. To apprehend the corporate investment behavior, the optimization of the compromise function is done from the point of view of the company (managers); that is, in relation to the variables that apprehend the actual amounts of the company's incomes, assets and expenses, i.e. G, r_E, I_E, r_D, I_D, B.[12]

[12] The manager considers the amounts that he earns or he effectively pay.

$$\begin{cases} \underset{G, r_E, I_E, r_D, I_D, B}{Max} \quad V(G\,; r_{nE}\,; I_E\,; r_{nD}\,; I_D; B) \\ u/c \\ \quad G \le (1-\tau)\left[\pi(K_0+I)-(\alpha_0 K_0+I_D)r_D-(K_0+I)\delta\right]-((1-\alpha_0)K_0+I_E)r_E \\ \quad I_D \le \varphi(r_{nD}; B) \\ \quad I_E \le \psi(r_{nE}) \\ \quad B \le B(I_D) \end{cases} \tag{34}$$

4. Business Investment Decision: resolution of the one period model

Business investment demand results from the resolution of the general compromise program (relation 34). For pedagogical reasons, the resolution of the capital compromise program is done here gradually by gradually removing the hypotheses of Orthodox theory. Thus, five main steps of the compromise program resolution are presented in order to be able to draw lessons on the consequences of each of the markets characteristics. It is about considering successively that:

(i) markets are imperfect and inefficient, and companies can only use loans to finance their investments;

(ii) the markets are imperfect and inefficient; companies can finance their investments only through the loanable funds market where they are faced funding constraints, without the possibility of using collaterals to improve their access to loans;

(iii) the markets are imperfect and inefficient; companies can finance their investments only through the loanable funds market where they are faced funding constraints, with the possibility of providing collaterals to improve their access to loans;

(iv) the markets are imperfect and inefficient; companies can finance their investments only through the loanable funds market where they are faced funding constraints, with the possibility of providing collaterals to improve their access to loans. Moreover, there is taxation in the economy;

(v) the markets are imperfect and inefficient; companies can use two categories of funds (equity and loan) to finance their investment projects; but for each source of funding, they are faced funding constraints; they can provide collaterals to access to loan. Moreover, there is taxation in the economy.

1. *Abandonment of the « perfect and efficient markets » hypothesis: effects of transaction costs on companies' investment decisions*

Under the assumption of "perfect and efficient markets", the working of the markets does not influence the investment decision of companies, only the law of supply and demand determines the quantity of investment and the interest rate. Thus, the transaction costs, which depend on the state of the markets functioning, would have no influence on the investment decision.

In reality, the markets are imperfect and inefficient, that is to say, the information is imperfect and the prices do not fully reflect the markets. Consequently, their functioning influences the decisions of economic agents. So, in this subsection, the neoclassical hypothesis of "perfect and efficient markets" is lifted. More specifically, it is about taking into account the imperfect and inefficient character of the

labor and capital markets in the analysis of the investment behavior of the companies. For now, it is considered that there are no funding constraints and that there is only one source of funding.

In accordance with the postulate of the general theory of the firm, the investment decision is the result of the capital compromise program which general form is given by relation 34. Under the assumption that there is only one financing source, that there are no taxation and funding constraints, this compromise program takes the form given by the optimization program (34a).

$$\begin{cases} Max\, V(G, I, r) \\ u/c \quad G \le \pi(K_0 + I) - (r + \delta)(K_0 + I) \end{cases} \tag{34a}$$

The first order conditions of this optimization program give the system of equations (35a). The first equation of this system is fundamental. Apart from the marginal rates of substitution (MRS) relating to the labor market and the capital market which appear in this equation, it remains equivalent to the first order condition of the investment decision of the neoclassical model.

$$\begin{cases} \dfrac{\partial F}{\partial K} + MRS_{IG} = (\delta + r) + \dfrac{\partial L}{\partial K} MRS_{L\pi} \\ TMS_{G/r} = K \\ G = \pi(K_0 + I) - (\delta + r)(K_0 + I) \end{cases} \tag{35a}$$

As shown in Box 1, the marginal rate of substitution of investment for retained earnings (MRS_{IG})[13] measures the marginal transaction cost of the investment borne by the company due to imperfections in the capital market. Likewise, the marginal rate of substitution of labor for gross profit ($MRS_{L\pi}$) measures the marginal transaction cost relating to the labor market.

Therefore, the first equation holds that the investment decision is such as the sum of the marginal productivity of capital ($(\partial F/\partial K)$) and the marginal transaction costs of investment in the capital market (MRS_{IG}) is equal to the sum of the user cost of capital ($\delta + r$) and the marginal transaction costs induced by investment in the labor market ($(\partial L/\partial K)MRS_{L\pi}$). Thus, according to this first equation, the investment decision is influenced by the transaction costs of the both markets of capital and labor.

According to this condition, the transaction costs of investment in the capital market (MRS_{IG}) induce inefficiency in corporate investment decision, resulting in overinvestment compared to the neoclassical optimum. The higher the degree of imperfection in the capital market, the higher the transaction costs and the higher the quantity of investment demand at which the investment project becomes profitable. For example, the higher the transaction costs of the stock market, the more it becomes inefficient to pay for a small number of shares and an unnecessary waste of time; you have to invest more to hope for net gains.

As for the transaction costs induced by investment in the labor market, the equation holds that they increase the capital user cost when the investment project requires an increase in the demand for labor ($\partial L/\partial K>0$) and thus leads to an under-investment compared to the neoclassical optimum. Conversely,

[13] $MRS_{IG} = \left(\dfrac{\partial V}{\partial I}\right) \Big/ \left(\dfrac{\partial V}{\partial G}\right)$

when the investment project induces a decrease in the labor demand ($\partial L/\partial K<0$), the labor market transaction costs cause overinvestment[14].

When the markets tend to be perfect and efficient, the first equation tends towards the neoclassical condition of the investment decision, since the respective capital and labor markets transaction costs tend towards zero. Thus, the neoclassical model of investment decision appears to be a particular case of the capital compromise model; it corresponds to the particular case where whatever the values of G, I and r, the capital compromise function can be written as an increasing function of G, i.e. the objectives of the stakeholders are fully independent; that is to say : V(G, I, r) = G.

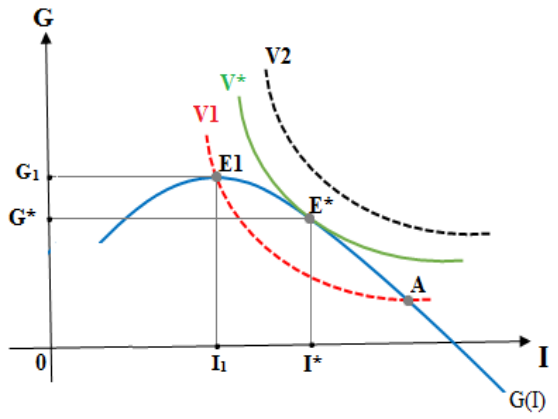
The graphical representation of first-order conditions (system of equations 35a) provides a better understanding of the effect of transaction costs on corporate investment decision. It is about representing retained earnings (G) of the company as an investment function (I) starting from the first and third equations of the first-order conditions given by the system of equations 35a. Also, the first equation is put in its most contracted form (system 36a) which holds that at "the optimum", the net profitability of the investment is equal to the opposite of the marginal transaction costs of the investment in the capital market i.e. the opposite of MRS_{IG}.

$$\begin{cases} \dfrac{\partial G(I)}{\partial I} = -\,MRS_{IG} \\ G(I) = \pi(K_0 + I) - (\delta + r)(K_0 + I) \end{cases} \qquad (36a)$$

As a reminder, for given values of G and I, the MRS_{IG} represents the slope of the capital compromise level curve at point (I, G). Likewise, the marginal retained earnings of investment represents the slope to the company's retained earnings curve at point (I, G). Thus, according to the system of equations (36a), the optimal compromise point corresponds to the point E* of coordinates (I*, G*) where the level curve of the capital compromise function is tangent to the retained earnings curve.

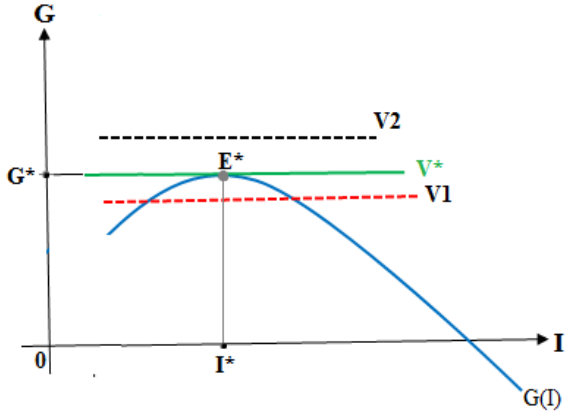
Graph 2: Demand for investments according to the nature of the labor and capital markets

Graph 2a: Case of imperfect and inefficient markets



Source : This Paper

Graph 2b: Case of perfect and efficient markets



Source : This Paper

When the markets are imperfect and inefficient (Graph 2a), the optimal compromise point of capital (E*) does not correspond to the neoclassical optimum (E1). Admittedly in point E1 of graph 2a, the retained earnings is maximum, however because of the transaction costs of investment in the capital market, point E1 does not allow reaching the highest level of compromise although it is technically accessible. In fact, on graph 2a, all the points on the retained earnings curve between E1 and A are much

[14] The term overinvestment here refers to a higher level of investment than the demand for optimal investment in the neoclassical sense. This remains in line with Josée's (2008) definition of overinvestment policy, which considers it to be an excessive investment policy compared to the level necessary to guarantee the maximum market value of the company. The under-investment policy corresponds to the opposite situation.

better capital compromises for the other stakeholders compared to point E1. So, lenders would not accept point E1 as a compromise point because they do not profit as they desired; thus, they would refuse to lend their funds for this investment project. For the level of capital compromise V1, lenders prefer the point A, a situation in which the managers are disadvantaged (low retained earnings); the managers would refuse to borrow under these conditions. So, the level of compromise V1 does not allow the interests of capital market participants to converge.

Following the same reasoning, regardless of each point of the retained earnings curve between E1 and E*, there is another point on the retained earnings curve that is a much better compromise for lenders. So, the point E* corresponds to the optimal compromise because there is no other technically possible point (located on the curve of retained earnings) giving a higher level of compromise; moreover, the interests are converging at this point.

The total transaction costs (TTCD) incurred by the company in its investment decision (E*) due to imperfections in the capital market is equal to G_1 minus G*, as shown by the calculation of the relation (37a) below.

$$TTCD = \int_{I_1}^{I^*} (MRS_{IG})dI = \int_{I_1}^{I^*} \left(-\frac{\partial G(I)}{\partial I} \right) dI = G(I_1) - G(I^*) = G_1 - G^* \qquad (37a)$$

When the markets are perfect and efficient (Graph 2b), the optimal capital compromise point (E*) corresponds to the neoclassical optimum. The fact that all market participants are fully informed and that each player makes its decisions in a rational way in the neoclassical sense, there is no room for negotiation and, thus the levels of satisfaction of the participants are independent of each other. This implies that the MRS_{IG} is equal to zero and, thus, the level curves of the capital compromise function are horizontal in the plane (I, G). So, like the previous case, the point E* corresponding to the optimal situation is the point where the retained earnings curve is tangent to the level curve of capital compromise, as shown in graph 2b.

Under the assumption of the convexity of the set of compromise possibilities in the capital market, analysis of the formula of transaction costs due to capital market imperfections borne by the company shows that:

(i) the marginal transaction costs of investment in the capital market are measured by the marginal rate of substitution of investment for retained earnings. In other words, the marginal transaction costs are equal to the ratio between “marginal compromise” of investment and “marginal compromise" of retained earnings. Thus, the lower the marginal compromise of retained earnings, the higher the transaction costs. This corresponds to reality because the more the expected retained earnings is full of uncertainty, the less an extra expected retained earnings would be important for the stakeholders; as a result, the lower the marginal compromise of retained earnings and, thus, the higher the transaction costs.

(ii) the relationship between marginal transaction costs and investment is negative[15]. That is to say, the more the quantity of investment increases, the more the transaction costs per unit of investment decrease. This supports the idea that financial intermediaries help reduce transaction costs in the capital markets (see Mishkin 2007).

[15] Remark that: $\frac{\partial MRS_{IG}}{\partial I} = \left(\frac{\partial^2 V}{\partial I^2}\frac{\partial V}{\partial G} - \frac{\partial^2 V}{\partial I \partial G}\frac{\partial V}{\partial I} \right) \Big/ \left(\frac{\partial V}{\partial G} \right)^2 \leq 0$

(iii) the relationship between the marginal transaction costs of capital market and retained earnings is positive[16]. This means in particular that the most risky investments are the most profitable. In another sense, this means that the more a economic agent accepts to invest effectively in information, the more he increases his "chance" of achieving a significant net gain.

(iv) the transaction costs of investment in the capital market lead to overinvestment while the transaction costs induced by investment in the labor market cause underinvestment in reference to the neoclassical optimum. So, taken together, the marginal transaction costs of labor market and capital market lead to underinvestment when transaction costs due to labor market are higher; they lead to overinvestment when transaction costs due to capital market are higher.

2. *Abandonment of the « absence of funding constraints » hypothesis: effects of funding constraints on companies' investment decisions*

In addition to the imperfect nature of the labor and capital markets, it is considered in this subsection that companies face more or less accentuated funding constraints. Indeed, in reality, funding is not available at will as it is advocated in neoclassical model. Several empirical studies have shown that, in particular, small and medium-sized enterprises generally face funding constraints in their investment decisions. Moreover, the existence of collateral systems in credit markets around the world proves that financing is not available at will for anyone who wishes to invest.

Thereby, in addition to taking into account the imperfection and inefficiency of the markets, it is about lifting the hypothesis "of absence of funding constraints", while considering that there is only one source of financing, without the possibility of providing collaterals.

In this case, the capital compromise program corresponds to the optimization program (34a) to which it is added the inequality of the funding constraint. This gives the program (34b) which indicates that the capital market participants seek to reach an optimal compromise under two constraints: the constraint of the profitability of the investment project and the constraint of financing. According to this second constraint, the investment made cannot exceed the financing supply which, in turn, depends on the interest rate r.

$$\begin{cases} Max\, V(G, I, r) \\ u / c \\ \quad G \le \pi(K_0 + I) - (r + \delta)(K_0 + I) \\ \quad I \le \varphi(r) \end{cases} \tag{34b}$$

The first-order conditions for this new capital compromise program are given by the system of equations (35b). Except for one term, the first equation of this system is identical to that of the system of equations (35a). The term (relation 36b) which is added to it represents the funding constraints. More specifically, this is the marginal opportunity cost induced by the funding constraints that companies face. The marginal opportunity cost being positive[17], this first equation clearly shows that the funding constraints induce an under-investment.

[16] Remark that: $\frac{\partial MRS_{IG}}{\partial G} = \left(\frac{\partial^2 V}{\partial G \partial I} \frac{\partial V}{\partial G} - \frac{\partial^2 V}{\partial G^2} \frac{\partial V}{\partial I} \right) \Big/ \left(\frac{\partial V}{\partial G} \right)^2 \ge 0$

[17] In the resolution of the program, it can be observed that the marginal opportunity cost of funding constraints is equal to Kuhn – Tucker's multiplier divided by the marginal compromise of retained earnings. The multipliers of Kuhn – Tucker been given positive or zero, then the expression 36b is positive or zero.

$$\begin{cases} \dfrac{\partial F}{\partial K} + MRS_{IG} = (\delta + r) + \dfrac{\partial L}{\partial K} MRS_{L\pi} + \dfrac{\left(D - MRS_{rG}\right)}{\dfrac{\partial \varphi}{\partial r}} \\ G = \pi(K_0 + I) - (\delta + r)(K_0 + I) \\ I = \varphi(r) \end{cases} \qquad (35b)$$

The marginal opportunity cost of funding constraints by debt (MOCFCD) (36b) increases with the stock of debt (D).[18] The higher the outstanding debt of the company (including the planned new loan), the more the lenders would be reluctant to finance the investment project.

$$MOCFCD = \frac{\left(D - MRS_{rG}\right)}{\dfrac{\partial \varphi}{\partial r}} \qquad (36b)$$

In addition, the marginal opportunity cost of funding constraints decreases with the interest rate elasticity of the financing supply. This means that funding constraints become more acute as the finance supply becomes increasingly inelastic at interest rates. This translates into reality because the finance supply becomes inelastic at the interest rate when loaners no longer have enough resources to finance companies and/or they are not willing to finance companies too because of a fairly unfavorable context (lack of confidence, high economic or political risks, etc.) even when the interest rate increases. In such a situation, whatever the level of interest rate, companies have difficulty in obtaining financing.

Also, the opportunity cost of funding constraints decreases with the marginal rate of substitution of interest rate for retained earnings (MRS_{rG}). It should be noted, that the MRS_{rG} measures the decrease of the retained earnings at the margin that managers would give up for an increase of one unit in the interest rate in order to get more loan. So, expression (36b) indicates that the more managers agree to pay lenders at high interest rates, the less the funding constraints will be strong. This is the same as saying that when lenders have relatively high interest rate requirements for managers, the higher the marginal opportunity costs of funding constraints.

Obviously, managers will be willing to pay high interest rates to their lenders for high return investment. When the profitability of the investment project is high, the retained earnings (G) of this project will be high; consequently, the marginal compromise of the retained earnings is relatively low (concave compromise function). So, the MRS_{rG} is relatively high and, as a result, the opportunity cost of funding constraints is low. Thus, the funding constraints are low for investment projects which profitability is higher than interest rate. This corresponds to one of the rating criteria for investment, which consists in comparing the internal rate of return (IRR) of the project to the interest rate.

Also, by differentiating the relation (36b) with respect to the interest rate (r) and to the retained earnings (G), it appears that the marginal opportunity cost of the funding constraints increases with the interest rate and decreases with expected retained earnings.

As before, the graphical representation of first-order conditions (system of equations 35b) provides a better understanding of the effect of funding constraints on corporate investment decision. It is about representing retained earnings (G) of the company as an investment function (I) starting from the equations of the first order conditions taken again in the system of equation 37b below.

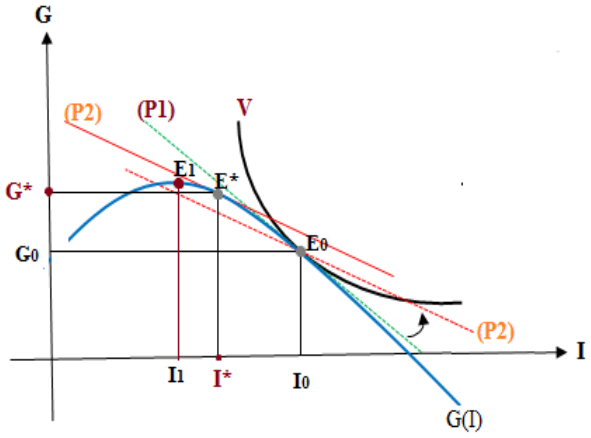
[18] For now, the analysis is doing with one source of financing: the credit.

$$\begin{cases} \dfrac{\partial G(I)}{\partial I} = -\left(MRS_{IG} - \dfrac{(D - MRS_{rG})}{\dfrac{\partial \varphi}{\partial r}} \right) \\ G = \pi(K_0 + I) - (\delta + r)(K_0 + I) \end{cases} \qquad (37b)$$

The first equation of the system (37b) shows that two situations are to be distinguished depending on whether the marginal transaction costs of investment is higher or lower than the marginal opportunity cost of the funding constraints. Indeed, if the marginal transaction costs of investment is greater than the marginal opportunity cost of funding constraints, then the optimal capital compromise corresponds to a point on the retained earnings curve with a downward slope. In other words, the optimal compromise point is on the retained earnings curve after the neoclassical optimum point and, thus, there is always an overinvestment compared to the investment level of the classical optimum. Conversely, if the marginal transaction costs of investment is less than the marginal opportunity cost of funding constraints, then the optimal capital compromise corresponds to a point on the retained earnings curve with an upward slope. In other words, the optimal compromise point is on the retained earnings curve before the neoclassical optimum point and, therefore, there is underinvestment compared to the investment level of the classical optimum.

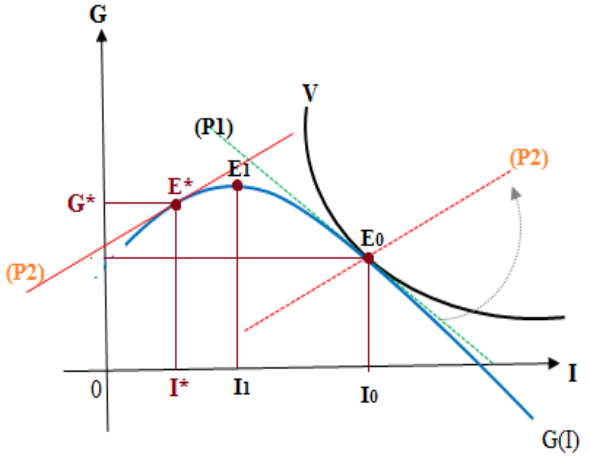
Graph 3: Demand for investment and funding constraints

Graph 3a: Cases where the marginal opportunity cost of funding constraints is lower than the marginal transaction cost of investment (relatively low constraints)



Source : This Paper

Graph 3b: Cases where the marginal opportunity cost of funding constraints is greater than the marginal transaction cost of investment (relatively strong constraints)



Source : This Paper

The two situations are illustrated by graphs 3. In these graphs, the optimal compromise without funding constraints corresponds to point E_0, where the level curve is tangent to the curve of retained earnings; the tangent corresponds to the slope (P1). Funding constraints reduce this slope by pivoting it from (P1) to (P2). The pivot angle of the tangent is greater or less depending on whether the funding constraints are more or less accentuated. So, the optimal compromise point with funding constraints is the point of tangency of the slope (P2) with the retained earnings curve (graph 3a and 3b).

Thus, in graph 3a, the funding constraints are relatively less accentuated (the slope (P1) pivots less), their negative effect on investment is relatively weak; there is an overinvestment ($I^* > I_1$). The decrease in investment due to funding constraints is equal to ($I_0 - I^*$).

On the other hand, in graph 3b, the funding constraints are relatively accentuated (the slope (P1) pivots more), their negative effect on investment is relatively high, so, there is an underinvestment ($I^* < I_1$). In this case, the decrease in investment due to funding constraints is equal to ($I_0 - I^*$) in Chart 3b.

At the end of this analysis highlighting the effect of funding constraints on the investment decision of companies, it appears that:

(i) funding constraints influence the investment decision of companies through the opportunity costs they generate;

(ii) funding constraints generate under-investment in companies;

(iii) the marginal opportunity cost of the funding constraints is equal to the ratio between, on the one hand, the outstanding debt of the company minus the marginal rate of substitution of the interest rate for retained earnings and, on the other hand, the marginal supply of financing from lenders with respect to the interest rate;

(iv) the marginal opportunity cost of funding constraints increases with the outstanding debt of the company;

(v) the marginal opportunity cost of funding constraints increases with the amount of investment financed by debt;

(vi) the marginal opportunity cost of funding constraints increases with the interest rate;

(vii) the marginal opportunity cost of the funding constraints decreases with the expected retained earnings.

3. *Abandonment of the « absence of collaterals » hypothesis: effects of financial guarantees on companies' investment decisions*

It was considered in the previous subsection that markets are imperfect and inefficient; companies face funding constraints, without the possibility of using collaterals to improve their access to loans. In this subsection, this assumption relating to collaterals is lifted and, thus, it is considered that it is possible to use collaterals to borrow. So, in accordance with the general compromise program (34), the financing supply now depends on the interest rate and the amount of collaterals. Also, the amount of collaterals obtained by lenders is less than or equal to the collaterals supply from the managers.

Therefore, the capital compromise program is given by the relation (34c) below which indicates that the capital market participants seek to reach an optimal compromise under three constraints, i.e. the profitability constraint of the investment project, the funding constraint and the collaterals constraint.

$$\begin{cases} Max\, V(G, I, r, B) \\ u/c \\ G \le \pi(K_0 + I) - (\delta + r)(K_0 + I) \\ I \le \varphi(r, B) \\ B \le B(I) \end{cases} \qquad (34c)$$

The first-order conditions for this new capital compromise program (with collaterals) are given by the system of equations (35c). Except for two terms, the first equation of this system is identical to that of the system of equations (35b). On the one hand, there is a term that is added to the left side of this equation and, on the other hand, the marginal opportunity cost of funding constraints by debt is now multiplied by a term which can be less than one.

$$\begin{cases} \dfrac{\partial F}{\partial K} + MRS_{IG} + \dfrac{\partial B}{\partial I} MRS_{BG} = (\delta + r) + \dfrac{\partial L}{\partial K} MRS_{L\pi} + \left(1 - \dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I}\right)\dfrac{(D - MRS_{rG})}{\left(\dfrac{\partial \varphi}{\partial r}\right)} \\ G = \pi(K_0 + I) - (\delta + r)(K_0 + I) \\ I = \varphi(r, B) \\ B = B(I) \end{cases} \quad (35c)$$

So, as might be expected, this new first-order equation shows that collaterals help reduce the opportunity cost of funding constraints by debt (relation 36c). According to this relationship, the more the financial supply is flexible with respect to collaterals and the supply of collaterals is positively sensitive to the amount of investment, the more the effect of collaterals on the opportunity cost of funding constraints by debt will be high. Ultimately, when the marginal supply of financing with respect to collaterals tends to one and the marginal supply of collaterals with respect to investment also tends to one, collaterals tend to eliminate the opportunity cost of funding constraints by debt.

$$MOCFCD(B) = \left(1 - \frac{\partial \varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{(D - MRS_{rG})}{\left(\dfrac{\partial \varphi}{\partial r}\right)} \quad (36c)$$

Furthermore, the first equation of the first order conditions holds that collaterals increase transaction costs in the capital market. With collaterals, the marginal transaction costs in the capital market (MTCFD) borne by the managers is equal to the marginal transaction costs due to the investment plus the marginal transaction costs due to the collaterals (relation 37c).

$$MTCFD(B) = MRS_{IG} + \frac{\partial B}{\partial I} MRS_{BG} \quad (37c)$$

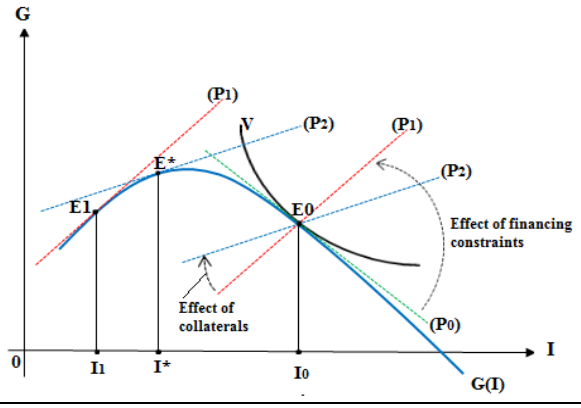
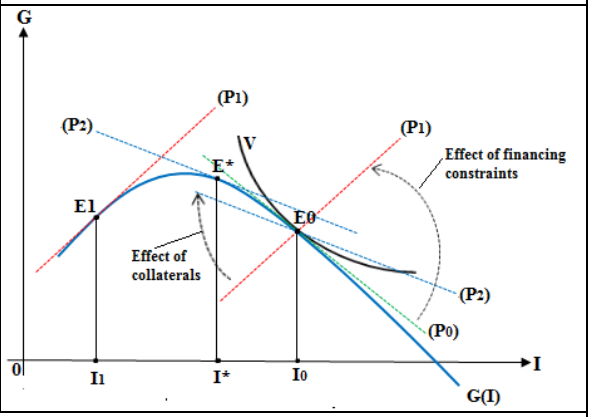
Ultimately, the collaterals provided by managers increase the level of investment by reducing funding constraints; but they increase the capital market transaction costs and thus increase the ineffectiveness of the investment decision.

The graphical representation of first-order conditions (system of equations 35c) provides a better understanding of the effect of collaterals on corporate investment decision. It is about, as previously, of representing retained earnings (G) of the company as an investment function (I) starting from the equations of the conditions of the first order taken again in the system of equation 38c.

$$\begin{cases} \dfrac{\partial G(I)}{\partial I} = -\left(MRS_{IG} + \dfrac{\partial B}{\partial I} MRS_{BG} - \left(1 - \dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I}\right)\dfrac{(D - MRS_{rG})}{\left(\dfrac{\partial \varphi}{\partial r}\right)} \right) \\ G = \pi(K_0 + I) - (\delta + r)(K_0 + I) \end{cases} \quad (38c)$$

The first relation of the system (38c) holds that two situations can arise depending on the sign of the difference between the marginal transaction costs and the opportunity cost of the funding constraints reduced by the collaterals. When this difference is negative (graph 4a), the funding constraints (without collaterals) are quite strong, but the collaterals system is relatively ineffective to reduce them. Without the collaterals, the funding constraints would be quite strong and, thus, lead to a point of compromise (point E1 of graph 4a) with a relatively low amount of investment (I_1 on graph 4a), where the marginal profitability of the investment remains high. On the other hand, collaterals help reduce these financial constraints, but their effect remains relatively low because of the low capacity/willingness of managers to provide collaterals in relation to their willingness to invest ($\partial B/\partial I$ is closer to 0) and/or the low availability of lenders to accept collaterals in exchange for large funding ($\partial\varphi/\partial B$ is closer to 0). Thus,

in this case, although the collaterals increase the level of financing and thus the level of investment, the collaterals system does not allow managers to have all the financing necessary to exploit all the investment opportunities. So, the collaterals system contributes to slightly increase investment (I_1 to I*), but it remains insufficient face to investment opportunities because the marginal profitability of capital always remains positive (E*); that means there are investment opportunities again.

Graph 4a : Less effective collaterals'system	**Graph 4b** : More effective collaterals' system
	

If the markets are imperfect and ineffective without funding constraints, the compromise would be at point E_0 with a quantity of investment equal to I_0 and a marginal profitability of investment corresponding to the slope (P_0). If there are funding constraints, the managers cannot exploit all the investment opportunities, then the marginal profitability of investment remains at a higher level (moving from P_0 to P_1), corresponding to the slope (P_1) giving the provisional compromise point E_1 with a quantity of investment equal to I_1. When the system of collaterals is less effective (graph 4a) or less effective (graph 4b), it alleviates funding constraints and provides managers less or more important new possibilities to exploit investment opportunities. As a result, the marginal profitability of the investment will decrease significantly or slightly (going from P_1 to P_2). Then, the final compromise point is the point E* (intersection between the slope P_2 and the retained earnings curve G (I)), with a quantity of investment equal to I*. Thus, on each of the two graphs, the effect of collaterals on investment is equal to (I* - I_1); it is relatively higher in the graph 4b.

Source: This Paper

When this difference between marginal transaction costs and the opportunity cost of funding constraints is positive (graph 4b), the funding constraints (without collaterals) would be low and/or the collaterals system is relatively more effective. The graph 4b illustrates a situation where funding constraints are strong with a relatively effective collaterals system. Indeed, without collaterals, the funding constraints would lead to a point of compromise (point E1 of graph 4b) with a relatively low quantity of investment (I_1 on graph 4b), where the marginal profitability of the investment remains high. Collaterals reduce these funding constraints because the capacity/willingness of managers to provide collaterals related to their willingness to invest is strong ($\partial B/\partial I$ is closer to one) and the availability of lenders to accept collaterals in exchange for funding is also high ($\partial\varphi/\partial B$ is closer to one). Thus, the system of collaterals allows managers to have all the financing necessary to exploit all investment opportunities, so that at the point of capital compromise obtained (i.e. E* in graph 4b), the marginal profitability of the investment is negative (downward slope). So, the collaterals system helps to increase investment (I_1 to I*) beyond the investment quantity of the neoclassical optimum.

In conclusion to this subsection, it can be noted that collaterals influence the investment decision of companies through two ways: the way of transaction costs and the way of opportunity cost of funding constraints.

(i) collaterals increase the transaction costs borne by companies in the capital market. In addition to the direct transaction costs of the investment act, there are transaction costs induced by the collaterals necessary to access financing. These transaction costs related to collaterals decrease with the amount of collateral and with the amount of investment;

(ii) collaterals reduce the opportunity cost of funding constraints. At this level, it appears that the effectiveness of collaterals depends both on the marginal supply of financing with respect to collaterals (i.e. the willingness of lenders to accept collaterals) and on the marginal supply of collaterals of the managers with respect to the quantity of investment (i.e. the managers willingness/ability to provide collaterals). The higher the two marginal supplies, the more effective the collaterals will be.

4. *Abandonment of the « no taxation » hypothesis: effects of taxation on corporate investment decisions*

So far, the reasoning has been done without taking taxation into account. The objective of this subsection is to integrate taxation into the model in order to understand its impact on business investment decisions. Thus, in addition to the assumptions of market imperfections and inefficiency, the existence of funding constraints and a system of collaterals, it is considered that: (i) there is a corporate income tax which rate is noted τ, (ii) interest due on loans is tax deductible and (iii) lenders pay taxes on interest received, the rate of which is noted t.

Thus, by negotiating the capital compromise, each participant considers the net amount of its earnings (without taxes). Thus, the lenders consider the net amount of interest received. Let us note r_n the interest rate after taxes, so: $r_n = (1\text{-}t)\, r$.

Therefore, the capital compromise program is given by the relation (34d) which indicates that the capital market participants seek to reach an optimal compromise under three constraints: the profitability constraint, the funding constraint and the collaterals constraint.

$$\begin{cases} Max\, V(G,\, I,\, r_n,\, B) \\ u \,/\, c \\ \quad G \le (1-\tau)\left[\pi(K_0 + I) - (\delta + r)(K_0 + I)\right] \\ \quad I \le \varphi(r_n,\, B) \\ \quad B \le B(I) \end{cases} \tag{34d}$$

The first-order conditions for this capital compromise program (with taxation) are given by the system of equations (35d).[19] The first equation of this system is almost identical to that of the system of equations (35c). The differences are at the level of the marginal transactions costs and the marginal opportunity costs of the funding constraints where the tax rates intervene. Before going ahead, it should be noted that if the tax rates are zero, the system (35d) gives the first order conditions of the model without taxation (system 35c).

$$\begin{cases} \dfrac{\partial F}{\partial K} + \dfrac{1}{(1-\tau)}\left(MRS_{IG} + \dfrac{\partial B}{\partial I} MRS_{BG}\right) = (\delta + r) + \dfrac{\partial L}{\partial K} MRS_{L\pi} + \left(1 - \dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I}\right) \dfrac{\left(D - \dfrac{1-t}{1-\tau} MRS_{r_n G}\right)}{(1-t)\left(\dfrac{\partial \varphi}{\partial r_n}\right)} \\ G = (1-\tau)\left[\pi(K_0 + I) - (r + \delta)(K_0 + I)\right] \\ I = \varphi(r_n,\, B) \\ B = B(I) \end{cases} \tag{35d}$$

[19] As a reminder, optimization is done from the point of view of the company (managers); that is to say in relation to the values (gross or net) of the variables which actually constitute incomes and expenses for the company (managers). Thus, the program (34d) is optimized with respect to G, I, r (not r_n) and B.

The first-order conditions of the tax-integrating model show that both the corporate income tax and the interest taxes borne by lenders affect the opportunity cost of funding constraints. As it can be observed with the new expression of the marginal cost of funding constraints (36d):

(i) the willingness of managers to accept a reduction in retained earnings for an extra unit of interest rates increases with the rate of corporation income tax. This is justified by the tax benefit of debt which makes it advantageous for the managers to accept higher and higher loan interest rates when the rate of corporate income tax increases;

(ii) the availability of lenders to finance corporate investment at a lower interest rate decreases with the tax on interest received. This is normal because (i) the net interest rate decreases with the tax and (ii) the supply of financing available to lenders decreases with the net interest rate.

$$MOCFCD(B\,;\,\tau\,;\,t)=\left(1-\frac{\partial\varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{\left(D-\frac{1-t}{1-\tau}MRS_{r_nG}\right)}{(1-t)\left(\frac{\partial\varphi}{\partial r_n}\right)} \qquad (36d)$$

$$MTCFD(B\,;\,\tau)=\frac{1}{(1-\tau)}\left(MRS_{IG}+\frac{\partial B}{\partial I}MRS_{BG}\right) \qquad (37d)$$

So, the tax on the interest of lenders contributes to increase the opportunity cost of funding constraints in the credit market, while the rate of corporate income tax contributes to reduce this opportunity cost thanks to the tax benefit of debt.

In addition, the corporate income tax increases the marginal transaction costs borne by managers in capital market (37d). The higher the rate of corporate income tax, the higher the transaction costs. This seems logical given that, for companies, any tax paid constitutes a cost related to a transaction or a set of transactions it would have carried out.

Let us note that the expressions of the marginal opportunity cost of funding constraints and the marginal transaction costs are different depending on whether the reasoning is done before or after corporate income tax. Expressions (36d) and (37d) give the marginal costs before income tax. Marginal costs after income tax are obtained by multiplying expressions (36d) and (37d) by (1-τ). In the following, it is noted respectively OCFCDn and MTCFDn the marginal cost of the funding constraints by debt after tax and the marginal costs of transaction in capital market after tax.

In summary, the introduction of taxation into the model holds that:

(i) the corporate income tax increases the capital transaction costs borne by companies and the willingness of managers to accept higher interest rates thanks to the tax benefit of debt. So, the opportunity cost of funding constraints decreases with corporate income tax.

(ii) the tax on lenders' interest increases the opportunity cost of funding constraints because it accentuates funding constraints by reducing lenders capacity/willingness to provide financial resources at reduced rates of interest.

5. *Abandonment of the hypothesis of « homogeneity of funding »*

In reality to finance their investments, companies can resort to several sources which cannot be considered as being analogous both in terms of costs and access constraints/facilities. These sources of

financing can be classified into two main categories: equity and loanable funds. Thus, in accordance with the general capital compromise program (34), the corporate investment decision results from optimizing the compromise between capital market players (managers, shareholders and lenders), under constraints of profitability, financing and collaterals (program 34e).

As a reminder, in the notations, the variables associated with each funding source are distinguished by the index E for equity and the index D for loanable funds. Also, note $\boldsymbol{\alpha}_0$ the share of debt in the initial capital stock (before investment).

$$\begin{cases} Max\, V(G\,;\, r_{nE}\,;\, I_E\,;r_{nD}\,;\, I_D;\, B) \\ u/c \\ \quad G \le (1-\tau)\left[\pi(K_0+I)-(\alpha_0 K_0+I_D)r_D-(K_0+I)\delta\right]-((1-\alpha_0)K_0+I_E)r_E \\ \quad I_D \le \varphi(r_{nD};B) \\ \quad I_E \le \psi(r_{nE}) \\ \quad B \le B(I_D) \end{cases} \tag{34e}$$

The first order conditions of the compromise program (34e) are presented in the system of equations (35e) where D and E represent respectively the total outstanding debt and the total equity of the company. The first equation of this system relates to the financing by equity, while the second equation relates to the financing by debt. The other equations express the saturation of the constraints of the compromise program.

$$\begin{cases} \dfrac{\partial F}{\partial K}+\dfrac{1}{1-\tau}MRS_{I_E G}=(\delta+\dfrac{r_E}{1-\tau})+\dfrac{\partial L}{\partial K}MRS_{L\pi}+\dfrac{E-(1-t_E)MRS_{r_{nE}G}}{(1-\tau)(1-t_E)\dfrac{\partial \psi}{\partial r_{nE}}} \\ \dfrac{\partial F}{\partial K}+\dfrac{1}{1-\tau}\left(MRS_{I_D G}+\dfrac{\partial B}{\partial I_D}MRS_{BG}\right)=(\delta+r_D)+\dfrac{\partial L}{\partial K}MRS_{L\pi}+\left(1-\dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I_D}\right)\dfrac{\left(D-\dfrac{1-t_D}{1-\tau}MRS_{r_{nD}G}\right)}{(1-t_D)\dfrac{\partial \varphi}{\partial r_{nD}}} \\ G=(1-\tau)\left[\pi(K_0+I)-(\alpha_0 K_0+I_D)r_D-(K_0+I)\delta\right]-((1-\alpha_0)K_0+I_E)r_E \\ I_D=\varphi(r_{nD};B) \\ I_E=\psi(r_{nE}) \\ B=B(I_D) \end{cases} \tag{35e}$$

Compared to the previous first-order conditions (35d), it appears that the second equation is identical to that studied in the previous sub-section, except that the index D clarifies that it relates to the credit market. However, the first equation is new; it relates to the equity market. It holds that the share of equity investment is such as the marginal productivity of capital plus the marginal transaction costs of investment financed by equity (MTCFE) is equal to the sum of the user cost of equity, the marginal transaction costs induced by investment in the labor market and the marginal opportunity cost of funding constraints in equity market (MOCFCE).

The marginal opportunity cost of funding constraints in equity market (36e) increases with (i) the total amount of equity (E) and (ii) the corporate income tax and dividend tax. It decreases with the elasticity of the supply of equity with respect to return rate of shares, as well as with the willingness of the managers to distribute an extra dividend when the rate of return demanded by the shareholders increases by one unit (MRS_{rnEG}).

$$MOCFCE = \frac{E - (1 - t_F) MRS_{r_{nE}G}}{(1 - \tau)(1 - t_E) \dfrac{\partial \psi}{\partial r_{nE}}} \tag{36e}$$

Also, the marginal transaction costs in the equity market including taxes (MTCFE) decreases with the quantity of investment financed by equity, but it increases with the corporate income tax. So, the tax takes its place as transaction costs to be paid by the companies.

$$MTCFE = \frac{1}{1 - \tau} MRS_{I_E G} \tag{37e}$$

With regard to the loanable funds market, the expressions of marginal opportunity cost of funding constraints (38e) and marginal transaction costs (39e) remain identical to those in the previous subsection. The marginal transaction costs decrease with the quantity of investment financed by debt and the quantity of collaterals; but they increase with the corporate income tax rate. As for the marginal opportunity cost of the funding constraints of the debt, it increases with the total outstanding of debt of the company, the rate of taxation of the interests of lenders and the elasticity of the supply credit with respect to the net interest rate. It decreases with the effectiveness of the collaterals, the rate of corporate income tax and the disposition of the managers to pay an extra interest.

$$MOCFCD = \left(1 - \frac{\partial \varphi}{\partial B} \frac{\partial B}{\partial I}\right) \frac{\left(D - \dfrac{1 - t_D}{1 - \tau} MRS_{r_{nD}G}\right)}{(1 - t_D) \dfrac{\partial \varphi}{\partial r_{nD}}} \tag{38e}$$

$$MTCFD = \frac{1}{1 - \tau} \left(MRS_{I_D G} + \frac{\partial B}{\partial I_D} MRS_{BG} \right) \tag{39e}$$

The three main equations relating to the first-order conditions of the general capital compromise program can be written in the form of expressions of the system (40e)[20].

The first equation indicates that the optimal quantity of investment financed by equity is such as the marginal profitability of investment financed by equity is equal to the spread between the marginal opportunity cost of the funding constraints and the marginal of transactions costs in the equity market.

$$\begin{cases} \dfrac{\partial G(I_E)}{\partial I_E} = \left[MOCFCEn(I_E\,;\, t_E\,;\, r_{nE}) - MTCFEn(G;\, I_E\,;\, r_{nE}) \right] \\ \dfrac{\partial G(I_D)}{\partial I_D} = \left[MOCFCDn(I_D;\, t_D;\, r_{nD};\, B) - MTCFDn(G;\, I_D;\, r_{nD};\, B) \right] \\ G(I_E; I_D) = (1 - \tau)\left[\pi(K_0 + I) - (\alpha_0 K_0 + I_D) r_D - (K_0 + I)\delta \right] - ((1 - \alpha_0) K_0 + I_E) r_E \end{cases} \tag{40e}$$

Likewise, the second equation indicates that the optimal quantity of debt-financed investment is such as the marginal profitability of the debt-financed investment is equal to the difference between the marginal opportunity cost of the funding constraints and the marginal transaction costs (excluding taxes on the profit) of the credit market.

[20] By similarity, it is noted MOCFCEn and MTCFEn the marginal costs after corporate income tax relating to the equity market.

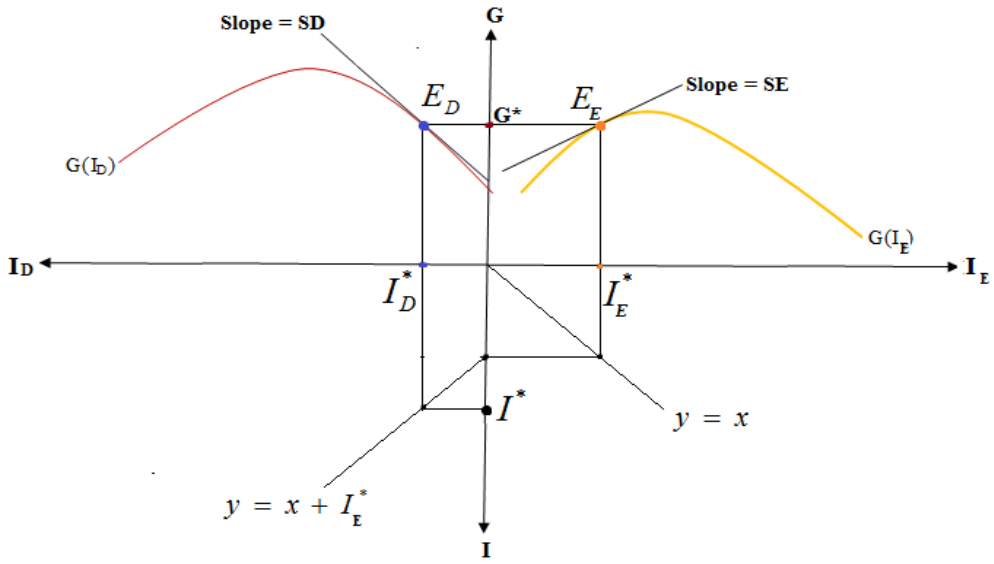
On the one hand, let us note by SE the difference between the marginal opportunity cost (after tax) of funding constraints and the marginal cost of transactions (after tax) in the equity market. On the other hand, it is noted by SD the difference between the marginal opportunity cost (after tax) of funding constraints and the marginal cost of transactions (after tax) in the credit market. In other words, let us pose $SE = [MOCFCEn - MTCFEn]$ and $SD = [MOCFCDn - MTCFDn]$.

Graphically, the first equation of the system (40e) indicates that the optimal decision of investment corresponds to the point E_E on the retained earnings curve on the equity market; at this point, the slope is equal SE (graph 5). The optimal demand for investment financed by equity is equal to I^*_E and the retained earnings is equal to G*.

Likewise, the second equation of the system (40e) indicates that the optimal decision of investment corresponds to the point E_D on the retained earnings curve on the credit market; at this point the slope is equal to SD (graph 5). The optimal demand for investment financed by debt is equal to I^*_D and the retained earnings is equal to G *.

Thus, with regard to funding constraints, transaction costs in the capital markets and profitability of capital, the total optimal demand for investment of the company is: $I^* = I^*_E + I^*_D$ as illustrated in the graph 5 below.

Graph 5: The optimal investment decision with two funding sources

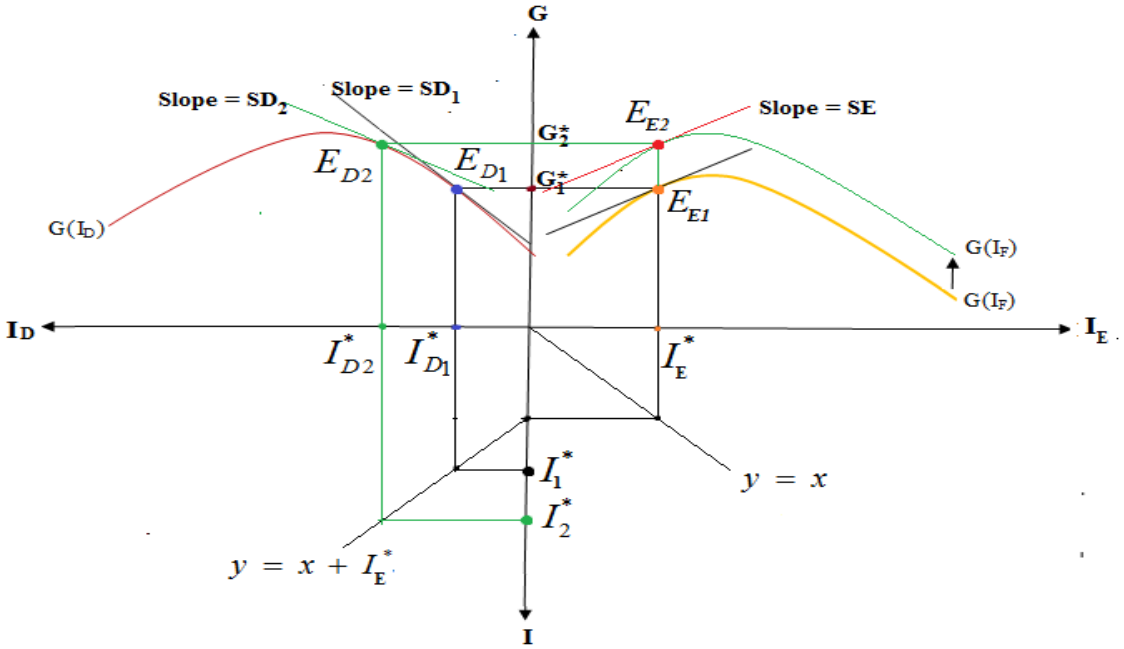


Source: This Paper

With regard to this graphical analysis, the question arises as to how the corporate investment decision changes when the funding constraints ease in one of the capital markets.

Let us start with an investment decision corresponding to what is illustrated in graph 5 and assume that the funding constraints in the credit market are easing. So, companies can now access more investment credit. As the profitability of the investment financed by debt remains high (the slope SD> 0), they will borrow and invest as much as possible and as much as the profitability remains positive.

Graph 6: Illustration of the effect of reducing funding constraints on the credit market



Source: This paper

On graph 6, the easing of the funding constraints and the reaction of companies to this event translate into the reduction of the slope SD which moves from SD_1 to the slope SD_2 on the retained earnings curve according to the financed investment by debt. So, on graph 6, debt-financed investment moves from I^*_{D1} to I^*_{D2}. This increase in debt-financed investment shifts the retained earnings curve of the equity market up: that is to say, for the same quantity of equity-financed investment, companies now make more retained earnings, as debt-financed investment has increased. Ultimately, the total investment moves from I^*_1 to I^*_2 and the retained earnings moves from G^*_1 to G^*_2 (graph 6).

With the example illustrated in graph 6, it appears that the financing structure has changed with the optimal quantity of investment. So, that raises the question on the relationship between the funding structure and the investment decision, which is examined in the next section. For clarity in the rest of the analysis, it is necessary to define the concepts of "total marginal cost of financing" and "total marginal transaction costs" which are used in the following sections.

Definition 1: For a given capital market, the term "total marginal cost of financing" means the sum of its interest rate after tax and its marginal opportunity cost of funding constraints. Thus, the total marginal cost of equity financing is equal to $r_E + MOCFCEn$ and the total marginal cost of financing from loanable funds is equal to $(1-\tau)r_D + MOCFCDn$.

Definition 2: For a given capital market, the term "total marginal transaction costs" means the sum of its marginal transaction costs of the investment and its marginal transaction costs of any collaterals. So, the total marginal transaction costs in the equity market is equal to *MTCFEn* and the total marginal cost of financing from loanable funds is equal to *MTCFDn*.

5. Investment decision and financing structure

One of the consequences of Modigliani-Miller's theorem (1958) is that under the hypothesis that the markets are perfect, efficient and without taxation, the optimal investment strategy is independent of the financing structure. Given that markets are imperfect, inefficient and that the taxation is omnipresent in

modern economies, the question arises how does the financing structure interact with the investment strategy?

To answer this question, let us call upon the both first order conditions equations given by the system (35e). By subtracting the first equation from the second and multiplying by (1-τ), the relation (41) is obtained. This relation holds that the optimal investment strategy is such as the total marginal costs of financing minus the total marginal costs respectively in the equity market and in the credit market are equal.

$$\left[r_E + MOCFCEn - MTCFEn\right] = \left[(1-\tau)r_D + MOCFCDn - MTCFDn\right] \qquad (41)$$

Note by MTCBDn, the marginal transaction costs (after tax) of the collaterals necessary to borrow. That is to say: $MTCBDn = \frac{\partial B}{\partial I_D} MRS_{BG}$.

The relation (42) is deduced from relation (41). It shows that the optimal investment strategy is such as the marginal rate of substitution of the investment financed by debt for the investment financed by equity (i.e. the managers preference for equity) is equal to one plus the ratio (i) of the difference between the total marginal costs of financing minus the marginal transaction costs of collaterals in the credit market and the total marginal costs of financing in the equity market, (ii) on the marginal transaction costs of investment financed by equity.

$$MRS_{I_D I_E} = 1 + \frac{\left[(1-\tau)r_D + MOCFCDn - MTCBDn\right] - \left[r_E + MOCFCEn\right]}{MTCFEn} \qquad (42)$$

On the basis of this result obtained by deduction, a theorem on the optimal investment strategy of the companies is stated.

Theorem 1: Corporate optimal investment strategy

Let us consider a market economy such as:

- *the set of compromise possibilities in the capital markets are convex;*
- *the gross operating surplus (gross profit) of companies is an increasing and concave function with the capital stock;*
- *the equity supply is an increasing and concave function with the net return rate required by the shareholders;*
- *the supply of loanable funds is an increasing and concave function with the net interest rate;*
- *the supply of collaterals of companies is an increasing and concave function with the amount of the investment credit.*

So, the optimal investment strategy $(I_E^*\ ;\ I_D^*)$ *is such as the total marginal financing costs minus the total marginal transaction costs respectively in the equity market and the credit market are equal; that is to say:*

$$\left[r_E + MOCFCEn - MTCFEn\right] = \left[(1-\tau)r_D + MOCFCDn - MTCFDn\right]$$

And, thus, at this optimum $(I_E^*\ ;\ I_D^*)$*, the companies' marginal preference for equity over loanable funds is equal to one plus the ratio (i) of the difference between the total marginal financing costs minus the marginal transaction costs of collaterals in loanable funds market and the total marginal financing costs of equity market, (ii) on the marginal transaction costs in the equity market; that is to say:*

$$MRS_{I_D I_E} = 1 + \frac{\left[(1-\tau)r_D + MOCFCDn - MTCBDn\right] - \left[r_E + MOCFCEn\right]}{MTCFEn}$$

From this theorem, it appears that the companies' preference at the margin for equity over loanable funds increases with the interest rate on loans and the marginal opportunity cost funding constraints by debt. Conversely, the companies preference at the margin for equity decreases with the yield required by the shareholders, the rate of corporate income tax, the marginal opportunity cost of funding constraints by equity, as well as with the marginal transaction costs in equity market.

One of the consequences of this theorem is that the financing structure influences the optimal investment strategy as long as the conditions in the capital markets are such as the difference between the two markets in total marginal financing costs minus the marginal transaction costs of possible collaterals is different from 0. To show this, let us examine how I^* changes when I_D^* varies by dI_D. To do this, let us pose $I_1^* = I_E^* + I_D^*$, the optimal quantity of initial investment.

First, consider that conditions in the capital markets are such as the difference of total marginal financing costs minus the marginal transaction costs of possible collaterals (i.e. the numerator of relation (42)) is strictly positive. So, at the optimum, the marginal rate of substitution of the investment financed by debt for the investment financed by equity is higher than one. So, it can be written that $MRS_{I_D^* I_E^*} = 1 + \varepsilon$, where $\boldsymbol{\varepsilon}$ is positive. Knowing that this MRS reflects the variation in equity required to keep the level of collective satisfaction (compromise) unchanged when the loan varies by one unit, then on the level curve of optimal compromise in the plan (I_F, I_D), it can be written that $MRS_{I_D^* I_E^*} = -dI_E / dI_D$. It is deduced that $dI_E = -(1+\varepsilon)dI_D$; which means that if I_D^* varies by dI_D, then I_E^* varies by $-(1+\varepsilon)dI_D$. So, the optimal quantity of total investment is $I_2^* = I_E^* - \varepsilon dI_D + I_D^* = I_1^* - \varepsilon dI_D$. Assuming that $\boldsymbol{\varepsilon}$ is positive, this result holds that the total investment increases if the variation dI_D of I_D^* is negative; it decreases if this variation is positive. That is to say if $dI_D < 0$, then $I_1^* < I_2^*$; on the other hand, if $dI_D > 0$, then $I_1^* > I_2^*$. So, the optimal quantity of investment changes when debt financing varies.

Secondly, let us consider that conditions in the capital markets are such as difference of total marginal financing costs minus the marginal transaction costs of possible collaterals (i.e. the numerator of relation (42)) is strictly negative. So, at the optimum, the marginal rate of substitution of the investment financed by debt for the investment financed by equity is less than one. It can therefore be written that $MRS_{I_D^* I_E^*} = 1 - \varepsilon$, where $\boldsymbol{\varepsilon}$ is positive. By reasoning similar to the previous one, it appears that $I_2^* = I_1^* + \varepsilon dI_D$. That is to say if $dI_D < 0$, then $I_1^* > I_2^*$; on the other hand, if $dI_D > 0$, then $I_1^* < I_2^*$.

In conclusion, when the conditions in the capital markets are such as the difference in their total marginal financing costs minus the marginal transaction costs of possible collaterals is not equal to zero, then any change in the investment financing structure leads to a change in the level of total investment demand. On this basis, a corollary relating to the relationship between the optimal investment strategy and the financing structure is deduced. However, before stating this corollary, the concepts of "iso-expensive" capital markets and "hetero-expensive" capital markets are introduced.

***Definition 3*:** Let $\Omega_E x \Omega_D$ be a subset of investment strategies (I_E ; I_D). The equity market and the loanable funds market are "iso-expensive" on the $\Omega_E x \Omega_D$ subset if, for any investment strategy (I_E; I_D) belonging to $\Omega_E x \Omega_D$, (i) the marginal transaction costs of investment in the two markets are equal and (ii) the total marginal financing costs minus the marginal transaction costs of possible collaterals in these two markets are equal. In other words, the two markets are said to be "iso-expensive" in $\Omega_E x \Omega_D$ if, for any strategy (I_F; I_D) belonging to $\Omega_F x \Omega_D$, on the one hand, $MRS_{I_D G}$ is equal to $MRS_{I_E G}$ and, on the other hand, $(1-\tau)r_D + MOCFCDn - MTCBDn$ is equal to $r_E + MOCFCEn$.

Note that if capital markets are perfect and efficient, there is no transaction cost, no funding constraints; and their net interest rates are equal; then they are iso-expensive.

***Definition 4*:** Let $\Omega_E x \Omega_D$ be a subset of investment strategies (I_E ; I_D). The equity market and the loanable fund market are "hetero-expensive" on the $\Omega_E x \Omega_D$ subset if there is at least one investment strategy (I_E ; I_D) belonging to $\Omega_E x \Omega_D$, such as (i) the marginal transaction costs of the investment of the two markets are different and/or (ii) the total marginal costs of financing minus the marginal transaction costs of any collaterals are different. In other words, the two markets are said to be "hetero-expensive" in $\Omega_E x \Omega_D$ if there is (I_E ; I_D) belonging to $\Omega_E x \Omega_D$, such as $MRS_{I_D G}$ is different from $MRS_{I_D G}$ and/or $(1-\tau)r_D + MOCFCDn - MTCBDn$ is different from $r_E + MOCFCEn$.

Also, if the equity and credit markets are iso-expensive within the meaning of definition 3, it appears, on the one hand, that $\left[(1-\tau)r_D + MOCFCDn - MTCBDn\right] - \left[r_E + MOCFCEn\right] = 0$ and, on the other hand, that the marginal rate of substitution of investment financed by debt for investment financed by equity is equal to one. Which means that $-\frac{dI_E}{dI_D} = 1$; that is to say $dI_E = -dI_D$. Thus, on the optimal capital compromise level curve, any variation in the investment financed by a given fund source leads to an equivalent but opposite variation in the investment financed by the other fund source. Thus, the increase in one leads to an equivalent decrease in the other; so, the total investment remains unchanged. Hence the corollary stated below.

Corollary 1: Optimal investment strategy and financing structure.

Let us consider a market economy such as:

- *the set of compromise possibilities in the capital markets are convex;*
- *the gross operating surplus (gross profit) of companies is an increasing and concave function with the capital stock;*
- *the equity supply is an increasing and concave function with the net return rate required by the shareholders;*
- *the supply of loanable funds is an increasing and concave function with the net interest rate;*
- *the supply of collaterals of companies is an increasing and concave function with the amount of the investment credit.*

If the credit and equity markets are hetero-expensive, then the optimal investment strategy depends on the financing structure.

A conversely, if the credit and equity markets are iso-expensive, then the optimal investment strategy is independent of the financing structure.

As indicated above, if the capital markets are perfect, efficient and without taxation in the sense of Modigliani-Miller (1958), they are iso-expensive. Also, when the markets are perfect and efficient and the marginal returns are diminishing, all assumptions of corollary below are confirmed. So, in application of this corollary, if the markets are perfect, efficient and without taxation, the optimal investment strategy is independent of the financing structure, as shown by Modigliani-Miller (1958).

In general, in reality, the capital markets (equity and loanable funds) are hetero-expensive because they have different modes of operation and accessibility conditions which induce different marginal costs. For example, for small and medium-sized enterprises (SMEs) for which managerial control of the company is an essential objective, the marginal transaction costs in the equity market are relatively higher than the marginal transaction costs in the credit market.

6. Investments, funding constraints and interest rates

As a reminder, both for neoclassical theory and Keynesian theory (Keynes 1936), investment decreases with the interest rate. However, the empirical literature has not yet succeeded in proving the existence of a negative relationship between investment and the interest rate. In view of these new theoretical results on the behavior of investment demand which take into account, among other things, transaction costs, funding constraints, collaterals and taxation, it seems necessary to re-examine the relationship (at theoretical plan) between the investment and the interest rate. Thus, this sub-section aims at examining the relationship between investment and interest rate of the credit market.

To do this, let consider the first order conditions (35d) relating to the loan market, the three main ones of which are taken up by the system of equations (35d bis). The study of the relationship between investment and the interest rate is not done here by considering "other things equal", but it is considered that the set of variables (G; I; r; B) could vary. So, the approach is to make the total differential of the first order conditions with respect to the four variables and to solve the system of equations obtained in order to express the variation (dI) of the investment as a function of the variation (dr) of interest rate.

$$\begin{cases} \dfrac{\partial \pi}{\partial K} + \dfrac{1}{(1-\tau)}\left(MRS_{IG} + \dfrac{\partial B}{\partial I} MRS_{BG} \right) - (\delta + r) - \left(1 - \dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I} \right) \dfrac{\left(D - \dfrac{1-t}{1-\tau} MRS_{r_n G} \right)}{(1-t)\left(\dfrac{\partial \varphi}{\partial r_n} \right)} = 0 \\ G = (1-\tau)\left[\pi(K_0 + I) - (r+\delta)(K_0 + I) \right] \\ B = B(I) \end{cases} \qquad \text{(35d bis)}$$

The resolve of the system of total differential of the first order conditions (35d bis) gives the equation (43) which expresses the variation of the demand for investment (dI) according to the variation of the interest rate (dr).

$$\left[A + \left(\frac{\partial \pi}{\partial K} - (\delta + r) \right) H \right] dI = M dr \qquad (43)$$

In this relation (43), A, H and M are parameters which depend on the capital compromise situation. Their expressions are given below:

$$A = \left[\begin{array}{l} \dfrac{\partial^2 \pi}{\partial K^2} + \dfrac{1}{1-\tau}\left(\dfrac{\partial MRS_{IG}}{\partial I} + \dfrac{\partial^2 B}{\partial I^2} MRS_{BG} + \dfrac{\partial B}{\partial I}\dfrac{\partial MRS_{BG}}{\partial I} \right) - \left(1 - \dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I} \right) \dfrac{1 - \dfrac{1-t}{1-\tau}\dfrac{\partial MRS_{r_n G}}{\partial I}}{(1-t)\dfrac{\partial \varphi}{\partial r_n}} + \dfrac{\dfrac{\partial B}{\partial I}}{1-\tau}\left(\dfrac{\partial MRS_{IG}}{\partial B} + \dfrac{\partial B}{\partial I}\dfrac{\partial MRS_{BG}}{\partial B} \right) + \\ \left(1 - \dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I} \right) \dfrac{\dfrac{\partial B}{\partial I}}{1-\tau} \dfrac{\dfrac{\partial MRS_{r_n G}}{\partial B}}{\dfrac{\partial \varphi}{\partial r_n}} + \left(\dfrac{\partial^2 B}{\partial I^2}\dfrac{\partial \varphi}{\partial B} + \dfrac{\partial B}{\partial I}\left(\dfrac{\partial^2 \varphi}{\partial B^2}\dfrac{\partial B}{\partial I} + \left(1 - \dfrac{\partial \varphi}{\partial B}\dfrac{\partial B}{\partial I} \right) \dfrac{\dfrac{\partial^2 \varphi}{\partial B \partial r_n}}{\dfrac{\partial \varphi}{\partial r_n}} \right) \right) \dfrac{\left(D - \dfrac{1-t}{1-\tau} MRS_{r_n G} \right)}{(1-t)\left(\dfrac{\partial \varphi}{\partial r_n} \right)} \end{array} \right]$$

$$H = \left(\frac{\partial MRS_{IG}}{\partial G} + \frac{\partial B}{\partial I}\frac{\partial MRS_{BG}}{\partial G} + \left(1 - \frac{\partial \varphi}{\partial B}\frac{\partial B}{\partial I} \right) \frac{\dfrac{\partial MRS_{r_n G}}{\partial G}}{\dfrac{\partial \varphi}{\partial r_n}} \right)$$

$$M = \left[\begin{array}{l} D\frac{\partial MRS_{IG}}{\partial G} - \frac{1-t}{1-\tau}\frac{\partial MRS_{IG}}{\partial r_n} + \frac{\partial B}{\partial I}\left(D\frac{\partial MRS_{BG}}{\partial G} - \frac{1-t}{1-\tau}\frac{\partial MRS_{BG}}{\partial r_n} \right) + \frac{\left(1 - \frac{\partial \varphi}{\partial B}\frac{\partial B}{\partial I}\right)}{\frac{\partial \varphi}{\partial r_n}}\left(D\frac{\partial MRS_{r_nG}}{\partial G} - \frac{1-t}{1-\tau}\frac{\partial MRS_{r_nG}}{\partial r_n} \right) + \\ 1 - \left(\frac{\partial^2 \varphi}{\partial B \partial r_n}\frac{\partial B}{\partial I} + \left(1 - \frac{\partial \varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{\frac{\partial^2 \varphi}{\partial r_n^2}}{\frac{\partial \varphi}{\partial r_n}} \right)\frac{\left(D - \frac{1-t}{1-\tau} MRS_{r_nG} \right)}{\left(\frac{\partial \varphi}{\partial r_n} \right)} \end{array} \right]$$

Under the assumption of the convexity of the set of capital compromise possibilities, it appears that the parameter H is positive. Also, the parameter M is positive if the funding constraints are strong enough. Also, if the funding constraints are strong enough, the net marginal profitability of capital is higher a threshold such as shown in the following inequality: $\left(\frac{\partial \pi}{\partial K} - (\delta + r)\right) \geq -\frac{A}{H}$. So, under such conditions, the relationship between investment and the interest rate is positive. Hence the theorem stated below.

Theorem 2 : funding constraints, investment and interest rates

Let us consider market economy such as:

- *the set of compromise possibilities in the capital markets are convex;*
- *the gross operating surplus (gross profit) of companies is an increasing and concave function with the capital stock;*
- *the supply of loanable funds is an increasing and concave function with the net interest rate;*
- *the supply of collaterals of companies is an increasing and concave function with the amount of the investment credit.*

Then there is a net marginal profitability threshold Z_g such as:

- *if the funding constraints are strong so that the net marginal profitability of the investment is greater than Z_g, that is to say $\left(\frac{\partial \pi}{\partial K} - (\delta + r)\right) > Z_g$, then, corporate investment increases with the interest rate.*
- *if the funding constraints are low so that the net marginal profitability of the investment is less than Z_g, that is to say $\left(\frac{\partial \pi}{\partial K} - (\delta + r)\right) < Z_g$, then, corporate investment decreases with the interest rate.*

The proof of this theorem results from the relation (43), which expresses the variation of the investment (dI) according to the variation of the interest rate (dr). Indeed, under the assumptions of the theorem and if the funding constraints are strong or weak, the parameter M of the relation (43) is positive and A is negative. So, the results of the theorem are obtained by setting Zg = -A/H.

Among other things, the theorem holds that in a situation of strong funding constraints, as long as the net profitability of the investment remains relatively high, the increase in the interest rate is favorable to business investment. This result, which goes against former theoretical results on investment behavior, remains consistent with recent empirical results on the direction of the relationship between the interest rate and investment (Naboulet and Raspiller 2006). Furthermore, this new theoretical result confirms the idea of Greenwald, Stiglitz and Weiss (1984) that market imperfections in capital market can lead to credit rationing so that the level of investment is determined by the credit supply, not by the cost of

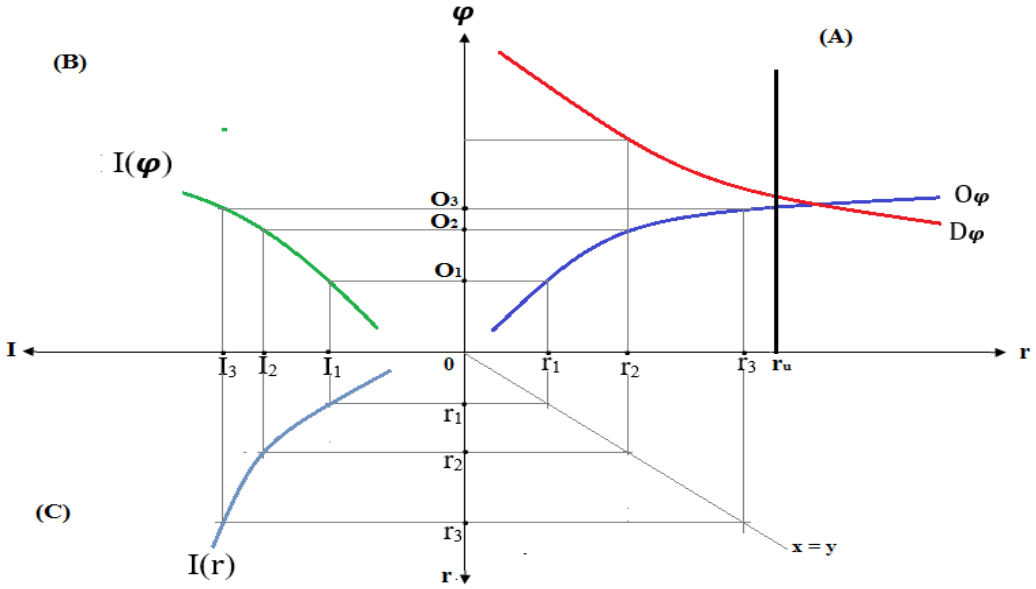
credit.

Also, credit rationing is associated with relatively high profitability and, therefore, under-investment as defined by Josée (2008). Indeed, a high level of net profitability of capital goes hand in hand with a greater demand for investment and, thus, a greater demand for financing to invest more in order to take advantage of this profitability of capital. So, if the profitability of capital remains relatively high, this reveals a weak capacity of companies to invest because of weak access to finance. Indeed, suppose that companies have the ability to access financing as they wish to invest while the profitability of capital is relatively high. Companies will invest to take advantage of the high level of profitability of capital; they will therefore request for funding. As they have access to the financing they want, the capital stock will increase and its net profitability will decrease until it is less than or equal to Zg. So, the abundant financing is linked with relatively low profitability of capital while an insufficient financing is linked with relatively high profitability.

As a result, if the profitability of capital remains relatively high, this reveals the low access of companies to financing to meet their demand for investment. In this case, an increase in the interest rate leads to an increase in the financing supply (incentive to save, possibility for bankers to resort to more expensive funds, etc.) and, thus, leads to an improving access to financing and an increasing of investment demanded. Hence when the profitability of capital is relatively high, corporate investment increases with the interest rate.

This result corresponds in particular to the fact that when investment is severely limited by the funding constraints, the level of investment is mainly determined by the financing supply, which, in turn, increases with the interest rate. Also, savings in a market economy increase with the interest rate; so, when the interest rate is very low or even negative, the level of savings remains low and, consequently, investment remains limited.

Graph 7: The demand for private investment when the demand for financing is rationed



Source : This Paper

Graph 7 illustrates the determination of the level of investment as a function of the interest rate when the demand for financing is rationed. The dial (A) of this graph represents the supply (Oφ) and demand (Dφ) of financing according to the interest rate. The financing supply function increases with the interest rate while the financing demand function decreases with the interest rate. However, due to constraints (insufficient loanable funds and/or high risks), the demand for financing remains higher than the supply of financing for any interest rate r lower than the usury rate r_u fixed by the regulation (graph 7). Then,

investment (the effective demand for investment) will adjust to the level of the financing supply with which it is increasing (dial (B)). As the financing supply increases with the interest rate, so investment increases with the interest rate (dial (C)).[21]

7. Business investment and expected retained earnings

This subsection aims to examine the relationship between corporate investments and expected retained earnings. To do this, as before, the total differentiation of the equations of the system (35d bis) is taken and the new system is solved in order to express the variation (dI) of the investment as a function of the variation (dG) of the expected retained earnings. Thus, the relation (44) is obtained.

$$\left[N+\left(\frac{\partial\pi}{\partial K}-(\delta+r)\right)R\right]dI=WdG \qquad (44)$$

In this relation (44), N, R and W are parameters which depend on the situation of capital compromise. Their expressions are given below.

$$N=\left[\begin{array}{l}\frac{\partial^2\pi}{\partial K^2}+\frac{1}{1-\tau}\left(\frac{\partial MRS_{IG}}{\partial I}+\frac{\partial^2 B}{\partial I^2}MRS_{BG}+\frac{\partial B}{\partial I}\frac{\partial MRS_{BG}}{\partial I}\right)-\left(1-\frac{\partial\varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{1-\frac{1-t}{1-\tau}\frac{\partial MRS_{r_nG}}{\partial I}}{(1-t)\frac{\partial\varphi}{\partial r_n}}+\frac{\partial B}{\partial I}\left(\frac{1}{1-\tau}\left(\frac{\partial MRS_{IG}}{\partial B}+\frac{\partial B}{\partial I}\frac{\partial MRS_{BG}}{\partial B}\right)\right)+\\ \frac{\partial B}{\partial I}\left(1-\frac{\partial\varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{\frac{\partial MRS_{r_nG}}{\partial B}}{(1-\tau)\frac{\partial\varphi}{\partial r_n}}+\left(\frac{\partial^2 B}{\partial I^2}\frac{\partial\varphi}{\partial B}+\frac{\partial^2\varphi}{\partial B^2}\left(\frac{\partial B}{\partial I}\right)^2+\frac{\partial B}{\partial I}\left(1-\frac{\partial\varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{\frac{\partial\varphi}{\partial r_n\partial B}}{\frac{\partial\varphi}{\partial r_n}}\right)\frac{\left(D-\frac{1-t}{1-\tau}MRS_{r_nG}\right)}{\left(\frac{\partial\varphi}{\partial r_n}\right)}\end{array}\right]$$

$$R=\frac{1}{D}\left[\frac{1-t}{1-\tau}\left(\frac{\partial MRS_{IG}}{\partial r_n}+\frac{\partial B}{\partial I}\frac{\partial MRS_{BG}}{\partial r_n}\right)-1+\left(1-\frac{\partial\varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{1-t}{1-\tau}\frac{\frac{\partial MRS_{r_nG}}{\partial r_n}}{\frac{\partial\varphi}{\partial r_n}}+\left(\frac{\partial^2\varphi}{\partial r_n\partial B}\frac{\partial B}{\partial I}+\left(1-\frac{\partial\varphi}{\partial B}\frac{\partial B}{\partial I}\right)(1-t)\frac{\frac{\partial^2\varphi}{\partial r_n^2}}{\frac{\partial\varphi}{\partial r_n}}\right)\frac{\left(D-\frac{1-t}{1-\tau}MRS_{r_nG}\right)}{\left(\frac{\partial\varphi}{\partial r_n}\right)}\right]$$

$$W=\frac{1}{1-\tau}\left[R-\left(\frac{\partial MRS_{IG}}{\partial G}+\frac{\partial B}{\partial I}\frac{\partial MRS_{BG}}{\partial G}+\left(1-\frac{\partial\varphi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{\frac{\partial MRS_{r_nG}}{\partial G}}{\frac{\partial\varphi}{\partial r_n}}\right)\right]$$

Under the assumption that the marginal transaction costs are diminishing, the parameters N, R and W are negative. It can be deduced that the investment increases with the expected retained earnings when the net marginal profitability of capital is above the threshold (- N/R). Conversely, the investment decreases with the expected retained earnings when the net marginal profitability is below the threshold.

In other words, if $\frac{\partial\pi}{\partial K}-(\delta+r)>-\frac{N}{R}$, then the company's investment will increase with the expected retained earnings and if $\frac{\partial\pi}{\partial K}-(\delta+r)<-\frac{N}{R}$, then the investment will drop with the expected retained earnings.

This theoretical result explains why companies have irregular investment behavior over time, with periods without investment, even if the expected retained earnings increases. Indeed, when the marginal

[21] The curve I (r) in the dial (C) is obtained by matching the interest rate of the dial A to the investment level of the dial B using the first bisector x = y in the last dial.

profitability of the investment is high (when new profitable investment opportunities arise), the companies concerned will invest, if they have the possibility, to sufficiently exploit these new opportunities and increase their retained earnings. However, after a period of massive investment, the net profitability of an extra investment becomes too low so that even if the expected retained earnings increases, it would no longer be profitable to invest in the short term, at least not until the next increase in net profitability of capital. As a result, companies whose funding constraints are relatively low and whose, in this way, have possibility of carrying out massive investment operations, will have more of an irregular investment behavior over time. For such companies, the investment behavior hardly depends on retained earnings. However, companies whose funding constraints are strong could have a less irregular investment behavior over time. Their investment behavior would depend on the retained earnings generated.

8. Conclusion

To compensate for the fragmentary nature of investment theory, this paper aims at developing a more global theoretical framework for corporate investment decisions, starting from the postulate of the General Theory of Firm (GTF) according to which company operates on the basis of the compromise between its stakeholders (managers, workers, shareholders and lenders). Two levels of compromise are distinguished: (i) the primary compromise which is related to the distribution of the added value of the company between the employer and the workers, (ii) the capital compromise which is related to the distribution of capital income between the managers (company), the shareholders and the lenders of the company.

In accordance with the intended objective, this paper is devoted to the capital compromise. The approach favored a deductive method based on the idea that the corporate investment decision is the result of the optimization of an implicit compromise function between the managers, the lenders and the shareholders, under profitability and funding constraints. It was implemented in a pedagogical and sequential manner by gradually introducing the main characteristics of the capital markets, i.e. imperfections, funding constraints, collaterals, taxation and the diversity of funding sources. Thus, several major theoretical results have been found.

Four lessons can be learned in relation with the optimal investment strategy. First, for each funding source, the corporate optimal investment strategy is such as the marginal productivity of capital plus the marginal transaction costs of the investment financed by this funding source is equal to the capital user cost plus the marginal transaction costs due to investment in the labor market and the marginal opportunity cost of funding constraints. Second, by distinguishing two main funding sources (equity versus loanable funds), the corporate optimal investment strategy is such as the total marginal costs of financing minus the total marginal costs of transaction in the equity market and in the loanable funds market are equal. Third, the optimal investment strategy depends on the financing structure when the capital markets are hetero-expensive; conversely, it is independent of the financing structure when the capital markets are iso-expensive. Fourth, whatever the conditions on the capital markets, the optimal financing structure is such as the company's marginal preference for equity over loanable funds is equal to one plus the ratio between, on the one hand, the difference between the two markets of the total marginal financing costs minus the marginal transaction costs of possible collaterals and, on the other hand, the marginal transaction costs of investment financed by equity.

Regarding the relationship between investment and the interest rate, it turns out that it is not monotonous. This relationship is positive when the funding constraints are strong and the net profitability of capital is high and above a threshold depending on the conditions on the capital markets. It is negative when the funding constraints are less accentuated and the net profitability of capital is low and below said threshold.

With regard to the quantity and fluctuations of investment, three lessons can be drawn. First, the irregular investment behavior of companies can be explained both by variations in the profitability of investment and by their ability or inability to carry out massive investment operations: the more the profitability of the capital decreases with the quantity of investment and the less the companies do not have funding constraints, the more the fluctuations of the investment will be strong. Second, funding constraints generate underinvestment through the opportunity cost they entail for companies in their investment decision. Conversely, the transaction costs linked to imperfections in the capital markets generate inefficiency (in the neoclassical sense) in the investment decision by promoting overinvestment. Third, collaterals have a double effect on corporate investment. On the one hand, collaterals reduce the opportunity cost of funding constraints by facilitating access to finance; which has a positive effect on investment. On the other hand, the collaterals contribute to reinforce the inefficiency of the investment decision by inducing the transaction costs for companies.

Thus, this paper opens up new perspectives for both empirical and theoretical researches on corporate investment decision. On a theoretical level, one of the main questions raised is that the theoretical results of this one-period model can be extended to explain the inter-temporal investment decision of companies: are the results of one-period model remain generally valid under the assumption that companies make intertemporels choices in terms of investment? A priori, the answer is yes; but it would be necessary to examine this question in depth to confirm and/or further specify certain results. On empirical level, the above results provide new lines of reflection or new theoretical considerations, on the one hand, to better understand and explain certain empirical results from the specialized literature, which do not corroborate the widely used and accepted theoretical results on the determinants of investment, and, on the other hand, to re-examine the determinants of corporate investment decision differently under more realistic assumptions.

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