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When Lower Interest Rates Slow Down Private Investment: Evidences from United States and Japan

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by

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Abstract:

Following the reflections in the General theory of the firm framework which showed that the relationship between investment and interest rate is not monotonous, this paper aimed (i) to carry out more precise theoretical investigations and empirical tests on the said relationship. Important theoretical advances have thus been made. First, the elasticity of investment with respect to lending interest rate is a function of several economic variables: (i) total of investment, (ii) interest rates in capital markets, (iii) expected retained earnings, (iv) respective stocks of capital financed by equity and on loans, (v) supplies of equity and loanable funds, as well as (vi) tax rate on business profits. Second, when firms are heavily rationed on funding, (i) the interest rate elasticity of investment is equal to 0 if the supply of loanable funds is insensitive to changes in the lending interest rate; (ii) it is positive if: (a) firms' aversion to paying interest is high, (b) the guarantee system is relatively efficient and (c) the supply of loans is relatively elastic with respect to the interest rate ; (iii) it is negative if the effectiveness of the guarantee system is quite limited and the supply of credit is inelastic with respect to the interest rate. Third, when the lending interest rate is relatively low, the interest rate elasticity of investment is negative if the expected rate of return on equity is sufficiently lower than the cash flow ratio of firms relative to capital stock. On the other hand, when the lending interest rate is relatively low, this elasticity would be positive during the period of strong speculation on the stock market. Econometric tests have confirmed this last result: the long run and short run elasticities of investment with respect to the lending interest rate were positive in United States and in Japan during the period 1996-2012 characterized by strong speculation on stocks markets. Thus, standardizing the use of the interest rate as a tool to boost private investment regardless of economic and financial conditions is inadequate and risky.

Résumé : Quand la baisse des taux d'intérêt freine l'investissement privé : Evidences empiriques aux Etats-Unis et au Japon

À la suite des réflexions dans le cadre de la Théorie générale de la firme qui ont montré que la relation entre l'investissement et le taux d'intérêt n'est pas monotone, ce papier s'est fixé pour objectif (i) de mener des investigations théoriques plus précises et de réaliser des tests empiriques sur ladite relation. Des avancées théoriques importantes ont été ainsi enregistrées. Premièrement, l'élasticité de l'investissement par rapport au taux d'intérêt sur les prêts est une fonction de plusieurs variables économiques : (i) l'investissement total, (ii) les taux d'intérêt sur les marchés des capitaux, (iii) l'autofinancement (profit net) escompté, (iv) les stocks respectifs des capitaux financés sur fonds propres et sur emprunts, (v) les offres de fonds propres et de fonds prêtables, ainsi que (vi) le taux d'impôt sur les bénéfices des entreprises. Deuxièmement, lorsque les entreprises sont fortement rationnées en matière de financement, (i) l'élasticité-taux d'intérêt de l'investissement est égale à 0 si l'offre de fonds prêtables n'est pas sensible aux variations du taux d'intérêt sur les prêts ; (ii) elle est positive si : (a) l'aversion des entreprises pour les intérêts à payer est élevée, (b) le système de garantie est relativement efficace et (c) l'offre de crédit est relativement élastique au taux d'intérêt ; (iii) elle est négative si l'efficacité du système de garantie est assez limitée et l'offre de crédit est peu sensible au taux d'intérêt. Troisièmement, lorsque le taux d'intérêt sur les prêts est relativement bas, l'élasticité-taux d'intérêt de l'investissement est négative si le taux de rendement escompté sur les capitaux propres est suffisamment inférieur au ratio de la capacité d'autofinancement des entreprises par rapport au stock de capital. En revanche, lorsque le taux d'intérêt sur les prêts est relativement bas, cette élasticité serait positive en période de fortes spéculations sur les marchés financiers. Les tests économétriques ont confirmé ce dernier résultat : les élasticités de long terme et de court terme de l'investissement par rapport au taux d'intérêt sur les prêts ont été positives aux Etats-Unis et au Japon au cours de la période 1996-2012 caractérisées par de fortes spéculations. La standardisation de l'utilisation du taux d'intérêt comme outil de relance de l'investissement privé quelles que soient les conditions économiques et financières est inadéquate et hasardeuse.

Keywords: investment, interest rate.
Mots clés : investissement, taux d'intérêt.
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Summary

1. Introduction

The interest rate is one of the most important variables of economic policy in general and of monetary policy. Used by central banks to regulate the liquidity of the economy, the interest rate has always been considered as a tool for reviving the economy, particularly through its relationship with investment. The concept of this tool of economic policy is based on the idea of the dominant economic thoughts (neoclassical and Keynesian) according to which the interest rate is the cost of capital, then the demand for capital (investment) falls when this cost increases ; it increases when this cost decreases. However, empirical studies have not been able to demonstrate such a relationship between the interest rate and investment. On the contrary, empirical studies have shown a positive relationship between the interest rate and investment (Naboulet A. and Raspiller S. 2006; Sharpe S. A. and Suarez G. A. 2014). Furthermore, Stiglitz and al. (1984) argue that imperfections of information in capital market can lead to credit rationing, so that the level of credit supply, not its cost, determines the level of investment. Thus, there would be a significant discrepancy between the facts and the theoretical results of the dominant thoughts on which the policy of stimulating investment through the reduction of the interest rate is based. Such a disconnect could lead to irrelevant decisions, the implementation of which could lead to undesired impacts, or even to an aggravation of the economic problems that the decision-makers thought they were solving.

This disconnect between the theoretical results of the dominant thoughts and the empirical evidence is linked to the fact that their basic hypotheses ignore, among other things, the imperfections and the compromises that are inherent in the relations between economic agents. Indeed, the idea that "the interest rate is the cost of capital and, thus, the demand for capital increases when this cost falls; it decreases when this cost increases" is a direct result of the neoclassical theory of corporate profit maximization in a framework of pure and perfect competition in which (i) economic agents are perfectly rational, (ii) each economic agent possesses the ability to access all the information in real time, (iii) the financing supply is available without limit and without constraint, (iv) the investment is reversible at will because the entrepreneur is only a tenant of the capital , (v) the enterprise has one aim, namely to achieve the maximum profit for the owners of the capital. Thus, despite its lack of realism, theoretical results based solely on this framework of pure and perfect competition continue to inspire certain economic policies throughout the world.

To overcome this shortcoming, a new theory of the firm has been proposed: General Theory of the Firm (Zerbo, A. 2016, 2018a and 2018b, Zerbo A. and Hien, L. 2019, 2020a). The General Theory of the Firm takes into account, among other things, all of the company's stakeholders, imperfections in the labor and capital markets, the compromises inherent in the relationships between economic agents, financing constraints and, thus, transaction costs and opportunity costs of financing constraints. Also, it postulates that, composed of several stakeholders, the firm is an entity that holds assets, signs contract, develops and manages specific know-how, promotes compromise between its stakeholders, produces goods and/or services in order to generate revenue for distribution to said stakeholders. From this more realistic theoretical framework, the general theory of the firm has established several new theoretical results both in relation to the labor market and in relation to the investment decision of companies (Zerbo, A. 2016, 2018a and 2018b, Zerbo A. and Hien, L. 2019, 2020a). One of his new theoretical results is that "the relationship between investment and the interest rate is not monotonous; it is more complex and can be increasing or decreasing". However, given the complexity of this relationship, this theoretical result remains quite general and does not provide sufficient detail on the conditions under which the relationship between investment and the interest rate would be positive and those under which it would be negative.

Then, this research aims to deepen the reflection through, on the one hand, more precise theoretical investigations to apprehend different economic and financial situations in which the relationship between investment and the interest rate is increasing or decreasing. On the other hand, it is a question of carrying out the econometric tests necessary to validate or invalidate some of these detailed theoretical results. These theoretical investigations are based on the results of the specification of the general theory of the firm (Zerbo A. and Hien, L. 2020b). We start from the results of this specification relating to

investment decision of the companies to (i) deduce the expression of the elasticity of the private investment with respect to lending interest rate and (ii) study the sign of the expression of this elasticity under different conditions. Next, empirical tests are carried out on the United States and Japan, the two developed countries for which data on private investment and lending interest rate are available in the "World Development Indicators" database of the World Bank over a longer period.

This paper is structured in four sections, namely, (i) a brief reminder of the general theory of the firm, (ii) the determination of the expression of the elasticity of investment with respect to interest rate, (iii) the study of the sign of this elasticity under different conditions and finally (iv) the empirical evidence.

2. Brief reminder of the General Theory of the Firm (GTF)

The basic idea of the General Theory of the Firm (GTF) is that in markets (labor, capital, goods and services, etc.), economic agents make deals or negotiate compromises between themselves. They negotiate, sign contracts, agreements and execute them in an environment characterized, among other things, by imperfect markets, information asymmetry and the existence of privileged relationships. Thus, the labor demand behavior of firms is dictated by the compromise process on the labor market, while their investment decision is determined by the compromise process on the capital markets.

1. *The basic postulate of the general theory of the firm*

The general theory of the firm is based on the idea according to which the company is an entity, composed of the employer or the team of managers, the workers and possibly the shareholders, which holds assets, signs contracts, develops and manages specific know-how, promotes compromise between stakeholders, produces goods and/or services to generate income which is distributed to said stakeholders. Also, the company possibly maintains privileged relations with banking and/or financial institutions, for the management of its cash flow and the financing of its investment projects.

Contrary to the neoclassical conception, the company does not only serve the interests of the employer, namely the maximization of profit. It aims to satisfy all stakeholders so that it performs sustainably in its creating value function. Although opposed, the interests of the stakeholders are interdependent. Indeed, as much as the employer wishes to make more net profit, it is in his interest that the employees feel satisfied so that labor productivity is high and that the shareholders and lenders also feel satisfied so that they continue to support the company's investment projects. Conversely, employees, to keep their jobs and have high salaries, have an interest in the employer making high profits, the shareholders and lenders being well remunerated so that they continue to support the business. Similarly, as much as the shareholders wish to have high dividends, they have an interest in ensuring that the company has the capacity to invest again, that the managers and employees are in satisfactory working conditions and that the lenders are suitably remunerated so that they continue to support the company's investment projects. As for the company's lenders, as much as they want to be remunerated at high rates, they have an interest in ensuring that the company can perform sustainably so that it can honor its commitments over time.

As a result, relationships between stakeholders are mutually conflicting. These mutually conflicting relationships force the company to operate based on compromises between the stakeholders (negotiations, agreements, contracts, conventions). Operating based on compromise does not exclude the adoption of opportunistic strategic or behavior by stakeholders. The asymmetry of information and the imbalance of bargaining power between the stakeholders favor such behaviors during the process and the execution of the compromise. Also, because of the changes that can occur in the relationships between the stakeholders, particularly at the informational and institutional level, the compromise is dynamic.

Thus, the state of compromise of the firm at a given moment depends not only on the institutional, legal and informational environment, but also on the economic, social and relational environment in which the company operates. For example, the position of each stakeholder in the negotiations depends

particularly on the fact that the economic environment offers or does not offer him other alternatives to achieve his interests (the fact of having other choices or not). Also, the state of relations of trust between the stakeholders (employer-lenders, employer-shareholders, employer-employees), the degree of rationality or altruism of the stakeholders, as well as the social relations between them influence the result of negotiations, namely the mutually advantageous situation.

In view of these elements, the general theory of the firm considers that the firm is characterized by an implicit function of compromise, namely "a subjective utility function ", which it seeks to optimize so that each stakeholder feels satisfied. This is in line with Williamson's principle of bounded rationality (Williamson 1975) according to which actors make choices that are intentionally rational, but inevitably limited because of the limits in their capacities to access and process information, as well as the limits imposed by the institutional, legal, relational and social environment. Obviously, just like a state of compromise, the function of compromise is characterized by the state of the institutional and legal, informational, economic, social and relational environment in which the company operates; therefore, it changes structurally with these elements. The measurable purposes of the stakeholders constitute the arguments of the compromise function. These include profit, wage rate, employment, investment, rate of return on capital, and bank guarantees.

Also, knowing that the principle of negotiation is to converge the positions of the stakeholders, intermediate solutions are preferred over extreme solutions, which implies that the set of compromise possibilities is convex, i.e. the compromise function of the firm is concave.

Moreover, in the operation of the company, two interdependent levels of negotiation are distinguished. On the one hand, there is the primary compromise which relates to the distribution of the added value between the wage bill (employees) and the gross operating surplus (employer who represents the owners of the capital). On the other hand, there is the capital compromise which concerns the distribution of income form capital between interest (lenders), retained earnings (managers) and dividends (shareholders).

2. *The process of the primary compromise in the company*

According to Zerbo (2016), the compromise between employers and employees (primary compromise) mainly concerns (i) the level of real wages (w/p), (ii) the level of employment (L) and (iii) the real gross profit (π), subject to the production possibilities constraint. Thus, given the institutional, informational, relational and social environment, the stakeholders seek to reach the optimal compromise, under the constraint of production possibilities.

Let U be the primary compromise function given by relation (1) and F be the production function of firms given by relation (2). Then, the firm's primary compromise program is given by relation (3).

$$U = U(\pi,\ L,\ w/p) \quad (1)$$

$$Y\ =\ F(K,\ L) \quad (2)$$

$$\begin{cases} Max\ U(\pi,\ L,\ w/p) \\ u/c \quad \pi + (w/p)L - F(K,\ L) \leq 0 \end{cases} \quad (3)$$

This primary compromise program determines, on the one hand, the wage bargaining process and, on the other hand, the labor demand behavior of firms. In fact, wage negotiation precedes the demand for work by companies, i.e. the stakeholders (employers and employees) agree on the remuneration of labor before its use. So, the business program can be distinguished in two phases. The first phase concerns the salary negotiation which makes it possible to determine the real salary level and the second phase concerns the determination of the labor demand.

Given the institutional, legal and informational environment of the labor market characterized by the primary compromise function U, as well as the quantity of labor necessary to produce a unit of good (L/Y), wage bargaining concerns the real wage (w/p) and the gross profit per unit of production (π/Y), under the constraint of distribution of the created wealth. Indeed, to set wages, the quantity of work per unit of production or, conversely, labor productivity is considered by the stakeholders as given, even if it is imperfectly known and is subject to moral hazard. Employers wish to remunerate this quantity of work per unit of production at a real wage level which would guarantee them both a high gross profit per unit of production (π/Y) and an acceptable level of work effort, while employees aim for a relatively high real wage level for the quantity of work per unit of production to be offered. Their interests being a priori divergent and interdependent, employers and workers are forced to negotiate to set the level of real wages.

Thus, from the primary compromise program of firms (relation 3), we deduce the wage bargaining program given by relation (4).

$$\begin{cases} \underset{\pi/Y,\, w/p}{Max}\ U\left(\frac{\pi}{Y},\ \frac{w}{p},\ \frac{L}{Y}\right) \\ s/c \quad \frac{\pi}{Y} + (w/p)\frac{L}{Y} \leq 1 \end{cases} \tag{4}$$

The first-order conditions give the system of equations (5) which states that the mutually advantageous real wage is such that the marginal rate of substitution (MRS) of profit per unit of output with respect to the real wage is equal to the quantity of labor per unit of output. The mutually advantageous agreement point ((w/p)*; (π/Y)*), solution of the system of equations (5), is such that the desire of employer to earn an extra penny on the real gross profit per unit of production is equal to the desire of an employee to earn an extra penny on the real wage.

$$\begin{cases} MRS_{\pi/w} = \frac{L}{Y} \\ \frac{\pi}{L} + (w/p) = \frac{Y}{L} \end{cases} \tag{5}$$

The resolution of these first-order conditions makes it possible to obtain the expression of the real wage as a function of the apparent productivity of labor (Y/L). By replacing, in the first equation of system (5), the gross profit ratio by its expression given by the income distribution constraint and by taking the total differential of this first equation, it emerges that the real wage increases with apparent productivity work. Thus, we can write relation (6) which expresses the real compromise wage as a function of labor productivity (Y/L).

$$(w/p) * = w_r\left(\frac{Y}{L}\right) \tag{6}$$

Once the real wage is fixed through the wage bargaining process, the firm maximizes the compromise function with respect to the levels of labor demand and real gross profit, under the constraint of distribution of the created wealth (relation 7). Indeed, for a fixed level of real wages (w/p), employers aim for a high level of real gross profit by minimizing, as far as possible, the total cost of labor (the level of employment); while employees aim for a high level of employment which would in particular make it possible to avoid layoffs and, at best, to reduce the workload per person. Their relationship being mutually conflictual, the stakeholders will enter negotiations to determine, based on the compromise, the level of employment and, thus, the real gross profit.

$$\begin{cases} \underset{\pi,\, L}{Max}\ U(\pi,\ L,\ w/p) \\ u/c \quad \pi + (w/p)L - F(K,\ L) \leq 0 \end{cases} \tag{7}$$

Solving this compromise program gives the results of the system of equations (8). Thus, given the imperfections of information, information asymmetries, the respective bargaining powers of the

stakeholders, labor legislation, contracts between them in the company and social relations, the optimal compromise (π* ; L*) in the labor market is the solution of the system of equations (8).

$$\begin{cases} \frac{\partial F}{\partial L} + MRS_{\pi/L} = w/p \\ \pi + (w/p)L = F(K,\ L) \end{cases} \qquad (8)$$

The first equation of system (8) gives the conventional curve of demand for labor, while the second equation expresses the technical curve of demand for labor. The mutually advantageous agreement (π*; L*) on the labor market represents the point of intersection of these two curves, i.e. the situation which is both technically and conventionally acceptable on the labor market. Under the assumptions of the concavity of the compromise function and the output function, this mutually advantageous point exists and is unique. Also, we show that the demand for labor is increasing with the real gross profit. Moreover, econometric tests carried out on OECD countries confirm that the demand for labor increases with gross profit (Zerbo 2018c).

In the first equation of first-order conditions (relation 8), the marginal rate of substitution of real gross profit with respect to labor ($MRS_{\pi/L}$) represents, on the one hand, the marginal transaction costs of the demand for labor supported by the employer because of labor market imperfections (existence of labor market institutions, imperfect information, information asymmetry, community relations). On the other hand, the $MRS_{\pi/L}$ reflects the degree of quantitative flexibility of labor. The more labor flexibility increases, the more the $MRS_{\pi/L}$ tends towards 0 and, thus, the system of equations (8) tends towards neoclassical conditions (maximization of profit) and the demand for labor of firms becomes less sensitive to changes in real gross profit. Conversely, the more the labor market becomes "rigid or imperfect", the more the $MRS_{\pi/L}$ increases and, thus, the demand for labor of companies becomes more sensitive to changes in real gross profit.

Solving the system of equations (8) gives the mutually advantageous employment level L* as a function of the real gross profit (π*) in the short run (relation 9) and as a function of the real wage and the capital stock in the medium and long term (relation 10).

$$L^* = L(\pi^*) \qquad (9)$$

$$L^* = L(w/p\ ;\ K) \qquad (10)$$

Thus, real gross profit is the determinant of employment in the short term. Under the hypothesis of the concavity of the compromise function and the production technology, we demonstrate that the level of employment increases with the real gross profit.

3. *The Process of Corporate Capital Compromise*

To invest, the company can use equity (shareholders) or loans (lenders). Thus, the investment decision of the company involves three types of actors: the team of managers, the shareholders, the lenders. This decision is the result of a compromise between these three stakeholders on the distribution of income generated by the investment project (interest, dividends and retained earnings) and the amounts of the investment by financing source.

In the capital compromise process relating to the investment project, (i) the management team expects a high retained earnings, (ii) the shareholders want more return on their shares and (iii) the lenders wish to benefit from high interest rates and substantial financial guarantees from the team of managers in relation to its commitments.

Then, there is an implicit compromise function, designated by "capital compromise function", which has as arguments the expected retained earnings (G), the return rate net of tax expected by the

shareholders (r_{nE}), the interest rate net of all taxes required by the lenders (r_{nD}), the respective shares of the investment to be financed by equity (I_E) and by loanable funds (I_D), the value of the financial guarantees required by the lenders (B). Denoted V, the capital compromise function is given by relation (11).

$$V = V(G\ ;\ r_{nE}\ ;\ I_E\ ;r_{nD}\ I_D\ ;\ B) \quad (11)$$

Thus, this capital compromise function takes into account (i) the purposes of the team of managers, namely the amount of the investment ($I_F + I_D$) and the expected retained earnings (G), (ii) those of the lenders which consist in granting the company a credit characterized by a net interest rate r_{nD}, an amount I_D and a guarantee B, as well as (iii) the purposes of the shareholders of the company which consists in placing funds of an amount I_F at a net rate of return r_{nF}.

For the team of managers, the ambition is to increase production capacities (capital stock) to generate a high flow of retained earnings which gives more possibilities of internal financing of the investment in the future. Let δ be the rate of depreciation of the firm's physical capital, τ the tax rate on profits and $\boldsymbol{\alpha}_0$ the share of debt in the capital stock K_0 at t=0. The retained earnings generated by the company is given by relation (12). Relation (13) recalls the expression of the gross profit which is involved in the expression of the retained earnings. The relation (14) indicates that the stock of capital is equal to the initial stock added to the total investment.

$$G(I_E, I_D) = (1-\tau)[\pi(K_0 + I_D + I_E) - (\alpha_0 K_0 + I_D)r_D - (K_0 + I_D + I_F)\delta] - ((1-\alpha_0)K_0 + I_E)r_E \quad (12)$$

$$\pi(K) = F(K,\ L) - (w/p)L \quad (13)$$

$$K = K_0 + I_D + I_E \quad (14)$$

As for the lenders, they have a financing supply function according to the project category (relation 15). The arguments of this financing supply function are, among others, the lending interest rate net of all taxes and the amount of the financial guarantees. The more the team of managers will be able to offer the necessary guarantees and/or remunerate the lenders at a high interest rate, the more the lender are willing to increase the supply of credit.

$$O_D = \phi(r_{nD}; B) \quad (15)$$

The amount of guarantees offered by the managers team to the lenders depends on the amount of the investment credit I_D (relation 16). It increases with the amount of the investment to be financed with loans.

$$O_g = B(I_D) \quad (16)$$

As far as shareholders are concerned, their financing supply depends on the net return they can expect from the investment (relation 17). Thus, the higher this rate of return net of tax, the more shareholders are willing to provide new capital for the investment project.

$$O_E = \psi(r_{nE}) \quad (17)$$

Then, based on these elements, the negotiation process leading to a compromise between the three stakeholders (managers, lenders, shareholders) relating to the investment project consists in optimizing the capital compromise function (relation 11) under the following constraints:

(i) the retained earnings generated by the project is greater than or equal to the minimum expected by the management team;

(ii) the share of the investment financed by borrowing is less than or equal to the supply of credit;

(iii) the share of the investment financed by equity is less than or equal to the supply of equity;

(iv) the amount of financial guarantees obtained by the lenders is less than or equal to the company's supply of financial guarantees.

Thus, the one-period capital compromise program is given by relation 18. To understand the firm's investment behavior, the optimization of the compromise function is done from the point of view of the team of managers ; that is to say in relation to the variables that identify the amounts of earnings, assets or expenditures for the company (team of managers), namely G, r_E, I_E, r_D, I_D, B.

$$\begin{cases} \underset{G,\, r_F,\, I_F,\ r_D,\, I_D,\, B}{Max} V(G\ ;\ r_{nE}\ ;\ I_E\ ;r_{nD};\ I_D;\ B) \\ u/c \\ \quad G \leq (1-\tau)[\pi(K_0+I)-(\alpha_0 K_0+I_D)r_D-(K_0+I)\delta]-((1-\alpha_0)K_0+I_E)r_E \\ \quad I_D \leq \phi(r_{nD};B) \\ \quad I_F \leq \psi(r_{nE}) \\ \quad B \leq B(I_D) \end{cases} \qquad (18)$$

The resolution of this optimization program makes it possible to highlight the general expressions of the marginal transaction costs and the marginal opportunity costs of the financing constraints on the capital markets.

In the equity market, the marginal transaction costs of equity funded-investment (MTCFE) are equal to the marginal rate of substitution of retained earnings with respect to equity funded-investment (MRS) divided by (1-τ), as shown by relation (19). Under the hypothesis of convexity of the set of compromise possibilities, the transaction costs of investing in equity decrease with the amount of the investment; they increase with the tax rate on profits and retained earnings.

$$MTCFE = \frac{1}{1-\tau} MRS_{G/I_E} \qquad (19)$$

The marginal opportunity cost of equity financing constraints is given by relationship (20). It increases with the total amount of equity of the company (K_E), the tax rate on profits. It decreases with the elasticity of the supply of equity in relation to its return rate, as well as with the marginal rate of substitution of retained earnings with respect to return rate on equity ($MRS_{G/rnE}$) which measures the readiness of the management team to pay a high return rate to shareholders.

$$MOCFCE = \frac{K_E - (1-t_E)MRS_{\frac{G}{r_{nE}}}}{(1-\tau)(1-t_E)\frac{\partial \psi}{\partial r_{nE}}} \qquad (20)$$

About the loanable funds market, the expression of the marginal transaction costs of the investment (MTCFD) is given by the relation (21). It consists of two types of marginal transaction costs: the marginal transaction costs of the act of investing (without financial guarantees) and the marginal transaction costs attributable to the mobilization of financial guarantees.

The marginal transaction costs of investing in the loanable funds market decrease with the amount of debt-financed investment and the amount of collateral; but they increase with the rate of tax on profits and the flexibility of the guarantee supply in relation to the amount of financing.

$$MTCFD = \frac{1}{1-\tau}\left(MRS_{G/I_D} + \frac{\partial B}{\partial I_D} MRS_{G/B}\right) \qquad (21)$$

As for the marginal opportunity cost of the financing constraints of the investment by borrowing (relation 22), it increases with the total outstanding financial debt of the company (K_D), the tax rate of the interests of the lenders and the elasticity of credit supply with respect to the net interest rate. It decreases with the efficiency of the financial guarantee system, the tax rate on profits and the willingness of the management team to pay a high interest rate, namely $MRS_{G/rnD}$.

$$MOCFCD = \left(1 - \frac{\partial \phi}{\partial B}\frac{\partial B}{\partial I}\right)\frac{\left(K_D - \frac{1-t_D}{1-\tau}TMS_{G/r_{nD}}\right)}{(1-t_D)\frac{\partial \phi}{\partial r_{nD}}} \quad (22)$$

The first-order conditions of the compromise program (18) are presented in the system of equations (23). The first equation of this system relates to the financing of investment by equity, while the second equation relates to the financing of investment by debt. The other equations express the saturation of the various constraints of the compromise program.

$$\begin{cases} \frac{\partial \pi}{\partial K} + MTCFE = (\delta + \frac{r_E}{1-\tau}) + MOCFCE \\ \frac{\partial \pi}{\partial K} + MTCFD = (\delta + r_D) + MOCFCD \\ G = (1-\tau)[\pi(K) - K_D r_D - \delta K] - K_E r_E \\ I_D = \phi(r_{nD}; B) \\ I_E = \psi(r_{nE}) \\ B = B(I_D) \end{cases} \quad (23)$$

Each of the first two equations states that the optimal quantity of investment financed through a given capital market is such that the marginal profitability of capital plus the marginal transaction costs of investment in said market equals the sum of the cost of use of this capital and the marginal opportunity cost of financing constraints on the said market.

From the first-order conditions (system 23), two theorems relating respectively to the optimal strategy of corporate investment and to the relationship between investment and the interest rate have been stated and demonstrated (Zerbo, A. and Hien, L. 2019, 2020a).

Regarding the optimal investment strategy theorem, it is applicable under two types of assumptions. The first hypothesis considers that the set of compromise possibilities is convex, i.e. the stakeholders negotiate with the aim of converging positions. The second category of assumptions relates to the concavity and growth of the gross profit function of firms, the supply functions of capital markets and the supply function of financial guarantees of firms in the loanable funds market with respectively the stock of capital, the interest rates and the amount of investment credit. In other words, it is considered that each of these functions is increasing with its respective argument, but with a decelerated rate of increase.

The theorem states, on the one hand, that, under the conditions described above, the optimal investment strategy $(I_E^* \ ; \ I_D^*)$ is such that the total marginal financing costs minus the marginal transaction costs on the market for equity and on the market for loanable funds are equal. That is, the optimal investment strategy $(I_E^* \ ; \ I_D^*)$ is such that relation (24) be verified.

$$\left[\frac{r_E}{(1-\tau)} + MOCFCE - MTCFE\right] = [r_D + MOCFCD - MTCFD] \quad (24)$$

On the other hand, the theorem states that at the optimum $(I_E^* \ ; \ I_D^*)$, the investor's marginal preference for equity over loanable funds (measured by the MRS_{I_E/I_D}) is equal to one (1) plus the ratio of (i) the difference between the total marginal financing costs minus the marginal transaction costs linked to any guarantees on the two capital markets and (ii) the marginal transaction costs of the equity-financed

investment.

$$MRS_{I_E/I_D} = 1 + \frac{[r_D + MOCFCD - MTCBD] \; - \; \left[\frac{r_E}{(1-\tau)} + MOFCE\right]}{MTCFE} \qquad (25)$$

That is, the investor's marginal preference for equity over loanable funds is given by relation (25) where the term MTCBD denotes the marginal transaction costs on the loanable funds market due to collateral financial.

Thus, one of the consequences of this theorem is that the financing structure influences the optimal investment strategy if the capital markets are hetero-expensive (a very common case). The optimal investment strategy is independent of the financing structure if the capital markets are iso-expensive (very exceptional case), i.e. if the net marginal costs of financing on the two markets are equal for all investment strategies (I_E ; I_D).

Modigliani-Miller (1958) theorem relating to the financing structure of firms is a corollary of this first theorem. Indeed, note that when the capital markets are perfect and without taxes and duties, the marginal transaction costs and the opportunity costs of the financing constraints are zero, moreover their interest rates are equal; then their net marginal financing costs are equal. Therefore, perfect markets in the sense of Modigliani-Miler are iso-expensive. Thus, as stated by Modigliani-Miller (1958), if the markets are perfect and without taxes and duties, then the investment policy of companies is independent of their financing structure.

The second theorem relates to the relationship between investment and the interest rate. Under the same assumptions as the first theorem, this second theorem states that investment is not a monotonic function of the interest rate. The relationship between the investment and the interest rate can be decreasing or increasing.

3. Expression of the elasticity of investment with respect to the interest rate

This section aims to establish the expression of the elasticity of investment with respect to the interest rate starting from the specification of the General Theory of the Firm (GTF) proposed by Zerbo and Hien (2020b). To do this, it first recalls the specification of the behavior functions of the GTF.

1. *The specification of the behavior functions of the GTF*

The general theory of the firm considers that firms have a production function depending on the stock of capital and labor, a compromise function between stakeholders that can be distinguished into a compromise function on the labor market and a compromise function of capital. In addition, they have a financial guarantee supply function which increases with the amount of credit requested. Consistent with the purpose of this article, these firm behavior functions are specified here as Cobb-Douglas functions.

Thus, the production technology of firms is given by relation (26) where a and b are positive parameters and less than 1. Thus, the technology function of firms is increasing and concave with each factor of production. For given levels of the factors of production K and L, the technology of production makes it possible to calculate the level of gross income (Y) that companies can generate.

$$F(K, L) = qK^a L^b \qquad (26)$$

The compromise function of the labor market (primary compromise) is given by relation (27) where α_1, α_2, α_3 are positive parameters whose sum is 1. Thus, the compromise function is increasing and concave with each argument, i.e. the set of compromise possibilities is convex.

$$U(\pi;\ w/p;\ L) = \pi^{\alpha_1}(w/p)^{\alpha_2}L^{\alpha_3} \quad (27)$$

The compromise function of capital is given by relation (28) where θ_1, θ_2, θ_3, θ_4, θ_5 and θ_6 are positive parameters whose sum is equal to 1. Thus, the compromise function of capital is increasing and concave with each argument; in other words, the set of compromise possibilities on the capital markets is convex.

$$V(G;\ r_{nD};\ I_D; B; I_E; r_{nE}) = G^{\theta_1} r_{nD}^{\theta_2} I_D^{\theta_3} B^{\theta_4} I_E^{\theta_5} r_{nE}^{\theta_6} \quad (28)$$

In these compromise functions, the coefficients α_i and θ_i measure the degree of "collective preference" granted to the variable concerned compared to the others. These coefficients depend on the bargaining power of the stakeholders, the institutional and legal framework of the markets concerned and the economic and social environment.

In the compromise function of the labor market (relation 27), for example the coefficient α_1 measures the degree of "collective preference" given to real gross profit over real wages and employment, given the respective bargaining powers and the institutional, legal and social environment. The higher the bargaining power of employers and/or the labor law is favorable to employers, the higher the coefficient α_1 will be relative to α_2 and α_3 which measures the respective degrees of collective real wage and employment preference. Conversely, the higher the wage bargaining power of the employees and/or the labor legislation is more favorable to wages than to profit and employment, the higher the coefficient α_2 will be with respect to α_1 and α_3.

Similarly, in the compromise function of capital (relation 28), for example the coefficient θ_2 translates the degree of "collective preference" given to the interest rate on loans. The higher the bargaining power of lenders and/or the credit legislation is more favorable to lenders compared to managers and shareholders, the higher the coefficient θ_2 will be compared in particular to θ_1 and θ_6 which reflect the degrees of collective preference given to the fact that companies generate retained earnings and to the fact of distributing dividends to shareholders. Another example, the more expensive the administrative procedures in terms of investment and/or the investment legislation is binding for investors, the higher the coefficients θ_3 and θ_5 will be. Also, the more complex and expensive the procedures for placing assets as collateral to mobilize credit, the higher the coefficient θ_4 will be. Finally, for example, the higher the illegal taking of corporate profits (bribes), the lower θ_1 will be.

Regarding the supply function of financial guarantees of companies, it is a function of the amount of the investment credit I_D as shown by the relation (29) where B_0 is a positive constant and the coefficient σ is positive and less than 1. Thus, the financial guarantee supply function of firms in the loanable funds market is increasing and concave.

$$B(I_D) = B_0 I_D^{\sigma} \quad (29)$$

In addition to these corporate behavior functions, the general theory of the firm considers that lenders and shareholders have financing supply functions. The supply of loanable funds depends on the interest rate net of taxes and on the amount of the financial guarantees. Thus, the supply function of loanable funds is given by relation (30) where φ_0 is a positive constant, a_1 and a_2 are positive coefficients less than 1.

$$\phi(r_{nD};\ B) = \phi_0\ r_{nD}^{a_1} B^{a_2} \quad (30)$$

About the equity supply function, it is depending on the return rate expected by the shareholders as shown by the relation (31) where ψ_0 is a positive constant and the coefficient a_3 is positive and less than 1.

$$\psi(r_{nE}) = \psi_0\ r_{nE}^{a_3} \quad (31)$$

Based on this specification, the application of theorems relating, on the one hand, to the optimal strategy of corporate investment and on the other hand to the relationship between investment and the interest rate has made it possible to obtain more precise results presented below.

2. *The elasticity of investment with respect to lending interest rate*

By applying the theorem on the optimal investment strategy to the specification of the GTF presented above, it appears that the optimal investment strategy of companies $(I_F^*\ ;\ I_D^*)$ verifies the relation (32). That is to say, the investment strategy of firms is such that the capital structure (K_E; K_D) ensures the equality of the total marginal costs of capital markets.

$$r_E + \frac{r_E K_E - \eta a_3 G}{a_3 \psi} \quad = \quad (1-\tau) r_D + \frac{(1-\tau) r_D K_D - \mu \beta G}{\beta \phi} \qquad (32)$$

$$Avec : \beta = \frac{a_1}{(1-\sigma a_2)}\ ; \quad \eta = \frac{(\theta_5 + \theta_6/a_3)}{\theta_1}\ ; \quad \mu = \frac{(\theta_2/\beta + \theta_3 + \sigma\theta_4)}{\theta_1}$$

Also, from this specification of the GTF, Zerbo A. and Hien L. (2020b) expressed respectively the change in debt-financed investment (dI_D) and the change in equity-financed investment (dI_E) as a function of changes in the lending interest rate (dr_D), in the expected rate of return on equity (dr_E), in the expected retained earnings (dG) and in the amortization rate (dδ). These expressions are given by the relations (33) and (34).

$$dI_D = \left(\frac{\lambda_1 \alpha(1-\tau) - A_E H_D}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E}\right) dr_D + \left(\frac{\lambda_1 (1-\alpha) + (A_E - 1) H_E}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E}\right) dr_E + \left(\frac{\frac{\lambda_1}{K} + \frac{\mu}{\phi} + (A_E - 1)\left(\frac{\mu}{\phi} - \frac{\eta}{\psi}\right)}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E}\right) dG - \left(\frac{(1-\lambda_1)(1-\tau)}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E}\right) d\delta \qquad (33)$$

$$dI_E = \left(\frac{\lambda_1 \alpha(1-\tau) + (A_D - 1) H_D}{T_F + \frac{r_F}{a_3 \psi} A_D}\right) dr_D + \left(\frac{\lambda_1 (1-\alpha) - A_D H_E}{T_E + \frac{r_F}{a_3 \psi} A_D}\right) dr_F + \left(\frac{\frac{\lambda_1}{K} + \frac{\eta}{\psi} + (A_D - 1)\left(\frac{\eta}{\psi} - \frac{\mu}{\phi}\right)}{T_E + \frac{r_F}{a_3 \psi} A_D}\right) dG - \left(\frac{(1-\lambda_1)(1-\tau)}{T_F + \frac{r_F}{a_3 \psi} A_D}\right) d\delta \qquad (34)$$

$$Avec : \alpha = \frac{K_D}{K}\ ; \quad \lambda_1 = \frac{a}{1-b}\ ; \quad A_D = 1 + \frac{\lambda_1 \beta \phi}{(1-\tau) r_D K_D}\left(\frac{G}{K} + (1-\alpha)[r_E - (1-\tau) r_D]\right)\ ;$$

$$H_D = \frac{\mu \beta G}{r_D \phi} + \frac{(1-\beta)}{\beta}\frac{(1-\tau) K_D}{\phi} + (1-\tau)\ ; \quad A_E = 1 + \frac{\lambda_1 a_3 \psi}{r_E K_E}\left(\frac{G}{K} + \alpha[(1-\tau) r_D - r_E]\right)\ ;\ H_E = \frac{\eta a_3 G}{r_E \psi} + \frac{(1-a_3)}{a_3}\frac{K_E}{\psi} + 1\ ;$$

$$T_D = \frac{\lambda_1}{K}\left(\frac{G}{K} + (1-\alpha)[r_E - (1-\tau) r_D]\right)\ ; \quad T_E = \frac{\lambda_1}{K}\left(\frac{G}{K} + \alpha[(1-\tau) r_D - r_E]\right)$$

By summing the relations (33) and (34), we obtain the relation (35) which expresses the variation of the total investment according to the respective changes in the lending interest rate (dr_D), in the expected rate of return on equity (dr_E), in the expected retained earnings (dG) and in the depreciation rate (dδ).

$$dI = \left(\frac{\lambda_1 \alpha(1-\tau) - A_E H_D}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E} + \frac{\lambda_1 \alpha(1-\tau) + (A_D - 1) H_D}{T_E + \frac{r_F}{a_3 \psi} A_D}\right) dr_D + \left(\frac{\lambda_1 (1-\alpha) + (A_E - 1) H_E}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E} + \frac{\lambda_1 (1-\alpha) - A_D H_E}{T_E + \frac{r_F}{a_3 \psi} A_D}\right) dr_E +$$

$$\left(\frac{\frac{\lambda_1}{K} + \frac{\mu}{\phi} + (A_E - 1)\left(\frac{\mu}{\phi} - \frac{\eta}{\psi}\right)}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E} + \frac{\frac{\lambda_1}{K} + \frac{\eta}{\psi} + (A_D - 1)\left(\frac{\eta}{\psi} - \frac{\mu}{\phi}\right)}{T_E + \frac{r_F}{a_3 \psi} A_D}\right) dG - \left(\frac{(1-\lambda_1)(1-\tau)}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E} + \frac{(1-\lambda_1)(1-\tau)}{T_E + \frac{r_F}{a_3 \psi} A_D}\right) d\delta \qquad (35)$$

From relation (35), we deduce the expression of the elasticity of investment (I) with respect to the lending interest rate (r_D) given by relation (36). [2]

$$e_{I/r_D} = \frac{r_D}{I}\left[\frac{\lambda_1 \alpha(1-\tau) - A_E H_D}{T_D + \frac{(1-\tau) r_D}{\beta\phi} A_E} + \frac{\lambda_1 \alpha(1-\tau) + (A_D - 1) H_D}{T_F + \frac{r_E}{a_3 \psi} A_D}\right] \qquad (36)$$

[2] On reminder, $e_{I/r_D} = \left(\frac{dI}{I}\right)/\left(\frac{dr_D}{r_D}\right) = \frac{dI}{dr_D} \times \frac{r_D}{I}$. Hence the elasticity of I with respect to r_D is equal to the coefficient of dr_D of relation (36) multiplied by the ratio of r_D and I.

By replacing the expressions of A_F, A_D, H_D, T_D and T_F in the relation (36) by their respective expression given in the sub-expressions of the relations (33) and (34), we obtain the detailed and complete expression of the elasticity of investment with respect to the lending interest rate given by the relation (37).

$$e_{I/r_D} = \frac{1}{I}\left[\begin{array}{l} \dfrac{\lambda_1\alpha(1-\tau)r_D - \left(\frac{\mu\beta G}{\phi} + \frac{(1-\beta)(1-\tau)r_D K_D}{\beta\phi} + (1-\tau)r_D\right)\left(1 + \frac{\lambda_1 a_3\psi}{r_F K_E}\left(\frac{G}{K} + \alpha[(1-\tau)r_D - r_E]\right)\right)}{\frac{\lambda_1}{K}\left(\frac{G}{K} + (1-\alpha)[r_E - (1-\tau)r_D]\right) + \frac{(1-\tau)r_D}{\beta\phi}\left(1 + \frac{\lambda_1 a_3\psi}{r_E K_E}\left(\frac{G}{K} + \alpha[(1-\tau)r_D - r_E]\right)\right)} + \\ \\ \dfrac{\lambda_1\alpha(1-\tau)r_D + \left(\frac{\mu\beta G}{r_D\phi} + \frac{(1-\beta)(1-\tau)K_D}{\beta\phi} + (1-\tau)\right)\left(\frac{\lambda_1\beta\phi}{(1-\tau)K_D}\left(\frac{G}{K} + (1-\alpha)[r_E - (1-\tau)r_D]\right)\right)}{\frac{\lambda_1}{K}\left(\frac{G}{K} + \alpha[(1-\tau)r_D - r_E]\right) + \frac{r_E}{a_3\psi}\left(1 + \frac{\lambda_1\beta\phi}{(1-\tau)r_D K_D}\left(\frac{G}{K} + (1-\alpha)[r_E - (1-\tau)r_D]\right)\right)} \end{array}\right] \quad (37)$$

This expression (37) shows that the elasticity of investment with respect to the lending interest rate is a function of (i) the total investment level, (ii) interest rates, (iii) the retained earnings expected, (iv) the respective levels of capital financed by equity and borrowings, (v) the respective levels of the supplies of equity and loanable funds, as well as (vi) the corporate income tax rate, as expressed by relation (38).

$$e_{I/r_D} = e_{I/r_D}(I;\ r_D;\ r_E;\ G;\ K_D;\ K_E; \phi\ ;\ \psi;\ \tau) \quad (38)$$

Also, this expression of the elasticity (relation 37) confirms that the relationship between the interest rate and the investment is not monotonous because it can be positive or negative depending on the values of the behavior parameters and the economic and financial environment. Thus, the next section analyzes the sign of the interest rate elasticity of investment under certain conditions.

4. Study of the sign of the interest rate elasticity of investment

In view of the complexity of the expression of the interest rate elasticity of investment, a study of the sign according to the usual method would be non-productive. To do this, the study of the sign of elasticity is made here in two specific cases, namely: (i) case where the lending interest rate tends towards zero, (ii) case where companies are faced with an insufficient supply of financing on both markets at the same time.

1. *Sign of the interest rate elasticity of investment at interest rate levels close to zero*

According to Zerbo A. and Hien L. (2020a), in a context where the interest rate is low, investment increases with the interest rate when (i) corporate credit demand is rationed, (ii) the supply of credit is elastic with respect to interest rate and (iii) the profitability of capital is relatively high.

To better understand this theoretical result, the limit of the interest rate elasticity of investment when the lending interest rate tends towards to 0 is calculated and its sign is studied. This limit is given by relation (39).

$$\lim_{r_D \to 0} e_{I/r_D} = \left(\frac{\mu\beta G}{\phi I\lambda_1(1-\alpha)r_E\left(\frac{G}{K} + (1-\alpha)r_E\right)}\right)\left[\lambda_1 a_3\psi\left((\alpha + (1-\alpha)^2)r_E - \alpha\frac{G}{K}\right) - r_E K_E\right] \quad (39)$$

From this expression (39), we observe that $\lim_{r_D \to 0} e_{I/r_D}$ is equal to the product of two factors. The first factor being positive, the sign of this limit is that of the second factor. So, the lending interest rate elasticity of investment can be positive or negative depending on the sign of the second factor.

a. Conditions under which the interest rate elasticity of investment is positive at lending interest rate levels close to zero.

If the second factor of relation (39) is positive, then $\lim_{r_D \to 0} e_{I/r_D} > 0$; which means that in this case, when the lending interest rate is very low (close to zero), the investment increases with the lending interest rate. Thus, this limit is positive if and only if the inequality (40) is verified.

$$\left([\alpha + (1-\alpha)^2]r_E - \alpha\frac{G}{K}\right) > \frac{r_E K_E}{\lambda_1 a_3 \psi} \qquad (40)$$

Inequality (40) holds if: (i) the expected return rate on the equity market (r_E) is higher than the expected retained earnings ratio with respect to capital stock (G/K), (ii) the profitability of the capital (λ_1) is higher than 0, (iii) the supply of equity is relatively elastic with respect to the expected net rate of return (a_3 relatively high compared to 0) and (iv) the supply of equity (ψ) is very high compared to the expected dividends ($r_E K_E$).

Thus, this inequality suggests that in periods of strong stock market speculation, when the expected rate of return on equities is beyond the financial capacities of companies, a low lending interest rates would have a negative effect on the private investment. Indeed, a fall in the lending interest rate (towards levels close to 0) associated with a high expected rate of return on the financial markets discourages the supply of loanable funds by encouraging savers and even banks to invest more and more on the stock market. If the interest rate on loans continues to fall while the expected rate of return on equities remains high or rises, sophisticated economic agents will borrow on the credit market to invest in the stock market in order to profit from the difference in rate. If this trend in interest rates persists, stock market speculation increases beyond the stock market value of listed companies, credit increases because it is mainly used to speculate on financial markets and, thus, corporate credit is tightening further while the supply of equity fuels more speculation than the investment projects of listed companies. As a result, private sector investment slows down.

In the recent past of the world, the period 1996-2012 was characterized by heavy speculation in the financial markets that led to the financial crises of 1998-2000 and 2007-2008. The total value of stocks traded in financial markets in the United States and Japan reached record levels of 321% and 146% of GDP respectively during this period (World Bank 2021). This enthusiasm of investors for shares would be linked to the fact that the expected return on these financial securities was high; but unfortunately, it was much higher than the real financial capacity of listed companies. At the same time, lending interest rates fell sharply to their lowest historical levels in these countries during this same period. In Japan, the lending interest rate has continuously fallen from 2.66% in 1996 to 1.40% in 2012, against an average rate of 7.30% and a minimum rate of 5.03% observed over the period 1960-1990 (World Bank 2021). In the USA, the lending interest rate has fallen from 8.27% in 1996 to 3.25% in 2012 (World Bank 2021).

Has this decrease in the lending interest rate to levels close to zero had a positive effect on investment? In view of the situation described above, all the ingredients seem to be present in these two countries during the period of recent crises for the interest rate elasticity of investment to be positive (i.e. inequality (40) is verified). This means that the fall in lending interest rates observed in the United States and Japan would have had a negative effect on investment. Therefore, it seems relevant to empirically test the following hypothesis: "the fall in the lending interest rate between 1996 and 2012 in the United States and Japan had a negative effect on private investment". Section 5 of this paper is devoted to the empirical verification of this hypothesis.

b. Conditions under which the interest rate elasticity of investment is negative at interest rate levels close to zero.

Contrary to the previous case, if the inequality (41) is verified, then $\lim_{r_D \to 0} e_{I/r_D} < 0$; which means that in this case, when the lending interest rate is very low (close to zero), the investment decreases with the interest rate. Inequality (41) holds in at least two distinct cases.

$$\left([\alpha + (1-\alpha)^2]r_E - \alpha\frac{G}{K}\right) < \frac{r_E K_E}{\lambda_1 a_3 \psi} \qquad (41)$$

First, inequality (41) holds if the expected equity return rate (r_E) is less than the ratio of expected retained earnings with respect to capital stock (G/K) so that the ratio between the expected return rate and the ratio of expected retained earnings, namely $(r_E/(\frac{G}{K}))$, less than the share α of debts in corporate capital.[3] This would be explained by the fact that if companies generate a relatively high retained earnings for low interest rate levels, they would be encouraged to reduce the initially planned volume of their investments in loanable funds when the lending interest rate increases, to adopt a self-financing strategy which can only partially cover this reduction in investment forecasts. So, investment falls when the lending interest rate rises.

Second, inequality (41) holds if the profitability of capital (λ_1) is not significantly different from zero in a context of moderate speculation on the stock markets (i.e. the supply of equity remains at reasonable levels). Obviously, if the profitability of capital is close to zero for lending interest rate close to zero, this means that the internal rate of return is also close to zero, so an increase in the lending interest rate leads to a negative internal rate of return. This leads companies to reduce the initially planned volume of their investments. So, investment falls when the interest rate rises.

2. *Sign of the interest rate elasticity of investment when the demand for financing is rationed*

The purpose of this subsection is to study the sign of the elasticity of investment with respect to the lending interest rate in a situation of strong rationing of the demand for corporate financing. To do this, we calculate the limit of this elasticity when the ratio of the supply of equity to equity-financed capital (ψ/K_E) and the ratio of the supply of credit to debt-financed capital (ϕ/K_D) simultaneously tend to zero. The expression of this limit is given by relation (42).

$$\lim_{\left(\frac{\phi}{K_D};\frac{\psi}{K_E}\right)\to(0\,;0)} e_{I/r_D} = \frac{\beta}{(1-\tau)r_D I}[(\beta-1)(1-\tau)r_D K_D - \mu\beta G] \qquad (42)$$

The relation (42) indicates that when the demand for corporate financing is heavily rationed, the elasticity of investment with respect to the lending interest rate can be positive, negative or equal to zero.

a. Conditions under which the interest rate elasticity of investment is zero when the demand for financing is rationed

When parameter β is equal to 0, we have $\lim_{(\phi/K_D\,;\,\psi/K_E)\to(0\,;0)} e_{I/r_D} = 0$. Also, β = 0 is equivalent to a_1= 0, a_1 being the elasticity of the supply of credit with respect to the interest rate.[4]

This means that in the event of an accentuated rationing of the demand for corporate financing, the elasticity of investment with respect to the interest rate would be equal to 0 if the supply of credit is not sensitive to changes in the lending interest rate. We deduce that in an economy characterized by a very

[3] In this case, the left term of the inequality (41) would be negative while the right one would be positive.

[4] On reminder, $\beta = \frac{a_1}{(1-\sigma a_2)}$ (see relation 32)

low savings capacities (individual and collective) and by imperfections[5] which induce an insensitivity of the supply of credit in relation to the lending interest rate and whose banking sector has a very low capacities to mobilize foreign resources, interest rate changes have no significant effect on private investment. Indeed, in such a context, increasing investment necessarily involves increasing the supply of credit, which is however insensitive to the lending interest rate.

This result confirms the thesis defended by Stiglitz and al. (1984) that capital market imperfections can lead to credit rationing, so that the level of credit supply, not its cost, determines the level of investment.

b. Conditions under which the interest rate elasticity of investment is positive when the demand for financing is rationed

When the ratios of the supply of equity to equity-financed capital (ψ/K_F) and of the supply of credit to debt-financed capital (ϕ/K_D) simultaneously tend to 0, the limit of rate elasticity of interest of the investment given by the relation (42) is positive if the inequality (43) is satisfied.

$$\frac{1-\sigma a_2}{a_1} < 1 - \frac{\mu G}{(1-\tau) r_D K_D} \qquad (43)$$

This inequality (43) holds if: (i) the financing constraints are relatively strong (i.e. the willingness of firms to pay interest is far lower than the interest expected by lenders (i.e. μG is sufficiently low compared to $r_D K_D$), (ii) the guarantee system is relatively efficient (i.e. σa_2 is close to 1) and (iii) the supply of credit is relatively elastic with respect to the interest rate (i.e. a_1 is positive and high). So, in a context of accentuated rationing of the demand for corporate financing linked to an insufficient supply of financing and fairly strong financing constraints (excluding guarantees) on the credit market, the interest rate elasticity of the investment is positive if the guarantee system is relatively efficient and the supply of credit is interest rate elastic.

In fact, if the supply of credit is elastic with respect to the interest rate, this means that lenders have the possibility of offering more and more financial resources to companies if the interest rate is more and more high. Such a situation may be due to the fact that the profitability of the capital is high, but the resources mobilized by the lenders are more and more expensive and/or the credit risk becomes more and more high for the segments of companies whose credit demand is rationed. As a result, an increase in the lending interest rate induces an additional amount of potential credit supply. If the guarantee system is effective, this potential supply of credit could be mobilized by companies to increase the volume of their investments given that their profitability is high. So, the investment increases with the increase in the lending interest rate which makes it possible to make available more credit supply.

c. Conditions under which the interest rate elasticity of investment is negative when the demand for financing is rationed

When the ratios of the supply of equity to equity-financed capital (ψ/K_F) and of the supply of credit to debt-financed capital (ϕ/K_D) simultaneously tend to 0, the limit of rate elasticity of interest of the investment given by the relation (42) is negative if the inequality (44) is satisfied.

$$\frac{1-\sigma a_2}{a_1} > 1 - \frac{\mu G}{(1-\tau) r_D K_D} \qquad (44)$$

This inequality (44) holds if the guarantee system is not efficient enough (i.e. σa_2 is close to 0) and the supply of credit is not very sensitive to the interest rate (a_1 close to 0) so that $1 - \sigma a_2$ is higher than a_1. Thus, in the event of an accentuated shortfall in the supply of financing, when the guarantee system is

[5] For example, a low banking rate, a disconnect between savings interest rates and lending interest rates and low access to banking information are imperfections in the banking market that negatively affect the sensitivity of supply of credit relative to the lending interest rate.

relatively inefficient and the supply of credit is not very sensitive to the interest rate, an increase in the lending interest rate has a negative effect on the investment.

In fact, if the supply of credit is not very sensitive to the changes in the lending interest rate, this means that an increase in the lending interest rate does not induce a significant amount of potential supply of credit. However, it accentuates financing constraints by increasing the gap between the total interest expected by lenders and firms' willingness to pay interest (firms' aversion to paying interest on loans). As financing constraints are accentuated for a level of financing supply that would not have increased, companies are revising their investment intentions downwards. So, the investment decreases with the increase in the lending interest rate.

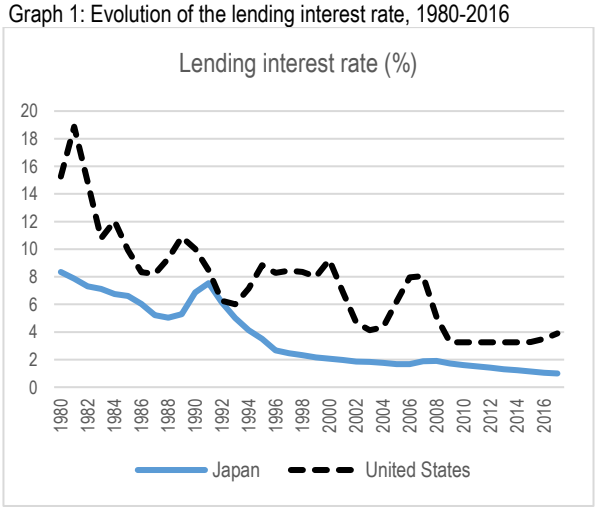
In summary, the relationship between investment and the interest rate is more complex than mainstream economic theories suggest. Indeed, rigorously, this analysis has shown, at the theoretical level, that there could be economic and financial situations in which the elasticity of investment with respect to the lending interest rate is positive. What is it in fact?

5. Empirical evidence: the interest rate elasticity of investment in the United States and Japan

The purpose of this section is to test the hypothesis formulated in the previous section, namely: "the fall in the interest rate on loans between 1996 and 2012 in the United States and Japan had a negative effect on the private investment". But before the empirical test, the evolutions of the lending interest rate, of the total value of stocks traded, and of the private investment is presented.

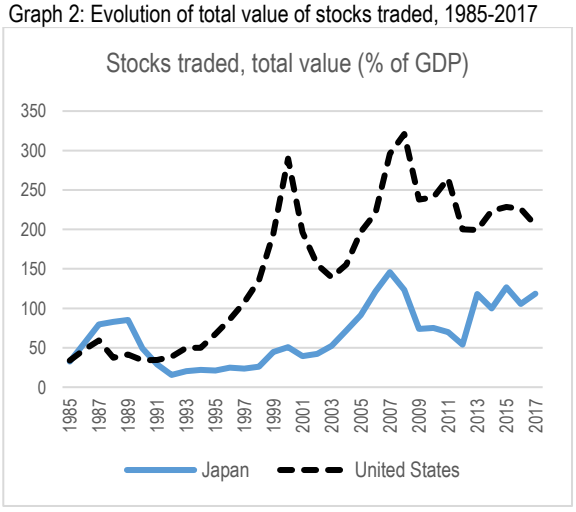
1. *Lending interest rate, total value of stocks traded and private investment in the USA and Japan*

Graph 1: Evolution of the lending interest rate, 1980-2016



Source: Based on WDI data, World Bank 2021

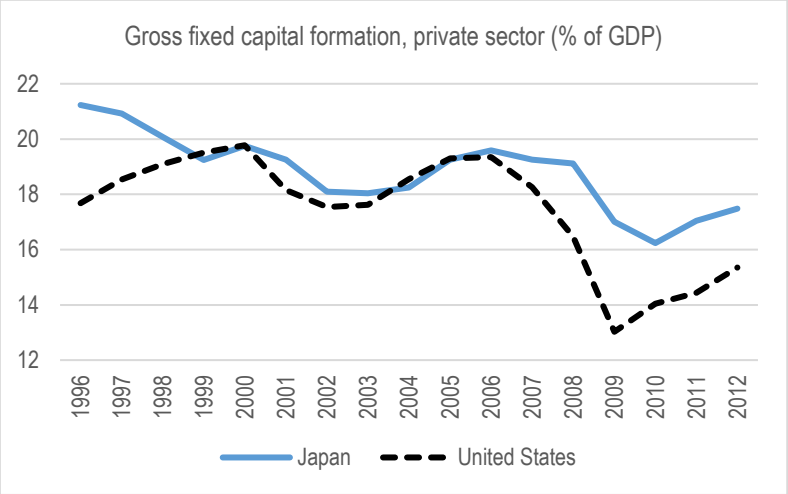
Graph 2: Evolution of total value of stocks traded, 1985-2017



Source: Based on WDI data, World Bank 2021

As a reminder, the 1996-2012 period was characterized by heavy speculation on the stocks' markets and financial crises in 1998-2000 and 2007-2008. The total value of stocks traded in the United States exceeded GDP in 1997 to reach a record level of 321% of GDP in 2008 (graph 2). At the same time, the lending interest rate fell in the USA to 3.25%, which is its lowest level since 1960 (graph 1). In Japan, the scenario was not too different: the total value of stocks traded increased from 24.6% of GDP in 1996 to 145.9% of GDP in 2007 and the lending interest rate, which had remained above 5% since 1960 fell to cross the floor of 3% in 1996 and reach 1.4% in 2012.

Graph 3: Evolution of private investment (in % GDP), 1996-2012



Source: Based on WDI data, World Bank 2021

Alongside the fall in interest rates, the private investment rate (graph 3) also fell in these countries between 1996 and 2012, representing an average annual growth rate of -1.2% in Japan and -0.9% in the United States. Thus, private investment rose from 21.2% of GDP in 1996 to 17.5% of GDP in 2012 in Japan and from 17.7% of GDP in 1996 to 15.3% of GDP in 2012 in the United States.

2. *Specification of models and data sources*

The empirical tests envisaged consist of the econometric estimation of panel models that relate private investment and the lending interest rate. Two categories of panel models are estimated. These are, on the one hand, two static panel models, respectively long-term and short-term, and, on the other hand, two dynamic panel models, also long-term and short-term.

a. Specification of static panel models

The first model is the static long-term panel model (equation 45) which expresses the private investment rate (I/Y) of period t as a function of the lending interest rate (r_D) of period t. In this long-term equation, I and Y denote private investment and GDP respectively, the indices i and t denote the country and period respectively, the parameter β_{LT} denotes the long-term elasticity of investment with respect to the lending interest rate, α denotes the specific effects of the model, u denotes the error term of the model.

$$Log\left(\frac{I_{it}}{Y_{it}}\right) = \beta_{LT} Log(r_{Dit}) + \alpha_i + u_{it} \qquad (45)$$

The second model is a static short-term panel model (equation 46) which expresses the annual change in the private investment rate of period t as a function of the annual change in the lending interest rate of period t and the private investment rate of period t-1. In this short-term equation, D denotes the operator of the annual change, the parameter δ_1 denotes the restoring force at equilibrium, β_{CT} denotes the short-term elasticity of investment with respect to the interest rate, θ denotes the specific effects of the model, ε denotes the error term of the model.

$$DLog\left(\frac{I_{it}}{Y_{it}}\right) = \beta_{CTd} DLog(r_{Dit}) + \delta_1 Log\left(\frac{I_{it-1}}{Y_{it-1}}\right) + \theta_i + \varepsilon_{it} \qquad (46)$$

b. Specification of dynamic panel models

In the dynamic panel models, it is assumed that private investment of period t is determined by its past values, as well as by the interest rate of period t and its past values.

Thus, in the dynamic long-term panel model, the lagged values of the logarithm of the private investment rate and the interest rate are introduced into equation (45). This gives equation (47) where the parameters γ_k are the coefficients of the dynamics of the private investment rate in relation to its past values in the long term. If there is at least one coefficient γ_k which is significantly different from 0, then private investment would behave dynamically in the long term.

$$Log\left(\frac{I_{it}}{Y_{it}}\right) = \sum_{k=1}^{P} \gamma_k \, Log\left(\frac{I_{it-k}}{Y_{it-k}}\right) + \beta_{LTd} Log(r_{Dit}) + \sum_{j=1}^{J} v_j \, Log(r_{Dit-j}) + \alpha_i + u_{it} \qquad (47)$$

Similarly, in the dynamic short-term panel model, the lagged values of the difference in the logarithm of the private investment rate are introduced into equation (46). This gives equation (48) where the parameters γ_h are the coefficients of the dynamics of the annual variation of the private investment rate in relation to its past values in the short term. If there is at least one coefficient γ_h which is significantly different from 0, then the annual change in private investment exhibits dynamic behavior in the short term.

$$DLog\left(\frac{I_{it}}{Y_{it}}\right) = \sum_{h=1}^{H} \gamma_h DLog\left(\frac{I_{it-h}}{Y_{it-h}}\right) + \beta_{CTd} DLog(r_{Dit}) + \delta_{1d} Log\left(\frac{I_{it-1}}{Y_{it-1}}\right) + \theta_i + \varepsilon_{it} \qquad (48)$$

The hypothesis of this empirical research is confirmed if and only if, on the one hand, the long-term elasticity (β_{LT} or β_{LTd}) is positive and, on the other hand, the short-term elasticity (β_{CT}) is positive and significantly different from 0 and the coefficient δ_1 of the restoring force at equilibrium is negative and significantly different from 0, or the short-term elasticity (β_{CTd}) is positive and significantly different from 0 and the coefficient δ_{1d} of the restoring force is negative and significantly different from 0.

c. Data sources and characteristics of the panel

The data used comes from the World Bank's "World Development Indicators database" downloaded in 2021. The variables used are: (i) the gross fixed capital formation (GFCF) of the private sector (% of GDP) and (ii) the lending interest rate (%). These data are available for the two countries (USA and Japan) for the years of the study period (1996-2012). Thus, the panel used is cylindrical. It has a total of 34 observations relating to two groups covering a period of 17 years (Table 1).

Table 1: Characteristics of the panel data used

	GFCF private/GDP	Lending interest rate
Number of observations	34	34
Number of groups	2	2
Number of periods	17	17
Overall average	18.133	3.973
Overall standard deviation	1.866	2.616
Inter-individual standard deviation (between)	0.964	2.920
Intra-individual standard deviation (temporal)	1.733	1.566

Source: Based on WDI data, World Bank 2021

3. *Econometric estimates of short run and long run interest rate elasticities of private investment*

The econometric estimates of the panel models presented above were carried out in accordance with the procedures for choosing the most suitable estimation method using the usual tests (Hausman test, LM Breusch and Pagan test) concerning the static models of panel. For the estimation of short-term and long-term dynamic models, the system generalized method of moments (GMM) is used. [6]

[6] There are two variants of the dynamic panel GMM estimators: the first difference GMM estimators and the system GMM estimators. Given the relatively small size of our sample, the system GMM method would be the most efficient (Blundel and Bond, 1998).

a. Long run lending interest rate elasticity of investment

The results of the estimations of the two long-term models are presented in Table 2.

Table 2: Results of long-term panel models estimates

Dependent Variable: Log(GFCF private /GDP) at t				
	Static panel model with fixed effects		**System Dynamic panel of long term**	
Explicative Variables	Coefficient	P>| t |	Coefficient	P>| t |
Log (Lending interest rate) at t	0.2788	0.000	0.2610	0.000
Log (Lending interest rate) at t-1	//////	//////	-0.2562	0.000
Log (GFCF private /GDP) at t-1	///////	///////	0.8237	0.000
Constant	2.5632	0.000	0.5075	0.007
Fixed effects *F(1 ; 31) =*	*81.893*	*0.000*		
Model quality statistics	*R²*	*0.7259*	*Nb. Obs = 32*	*Prob>Wald chi2 = 0.000*
	Adjusted R²	*0.7082*	***Instrumental variables:***	
	Prob>F(1;31)	*0.000*	***Differentiated equation***	
	Nb. Obs.	*34*	**GMM-Type**: L(2/.) .Log(GFCF private/GDP) at t	
Hausman test	*Chi2*	*32.59*	**Standard**: D.Log (interest rate) at t ; D.Log(interest rate) at t-1	
	Prob>chi2	*0.000*	**Level equation**	
			GMM-Type: D.Log(GFCF private/GDP) at t-1	
			Standard: Constant	

Source: Based on WDI data, World Bank 2021

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For estimating the long-term static model, the results of the Hausman test (Table 2) indicate that the fixed-effects model is better suited than the random-effects model (Prob<0.05). Therefore, the Ordinary Least Squares method is applied with the country dummy variables. Fischer's statistics show that the long-term static model is globally significant (Prob(F)<0.01) and the adjusted R^2 indicates that this model has an explanatory power of 70.8% of the variance of the private investment rate from Japan and the United States. Thus, in this static model, the long-term elasticity of the private investment rate with respect to the lending interest rate is significant at the 1% threshold and it is positive (0.2788).

This result is confirmed by the dynamic long-term panel model (Table 2). First, the estimates show that this model is globally significant (Prob (Wald's Chi2) < 0.01). Also, the coefficient of the private investment rate at t-1 is significant at the 1% level, thus confirming the autoregressive dynamic nature of the investment rate in the long term. The elasticities of the private investment rate with respect to the lending interest rate of the current period (0.2610) and to the lending interest rate of the past period (-0.2517) are significant at the 1% level and respectively positive and negative. This means that a 1% drop in the lending interest rate at t would have induced a 0.26% drop in the investment rate at t and a relatively smaller 0.25% increase in the investment rate at t+1. Thus, a 1% drop in the interest rate at t would have induced overall stagnation (or even a drop) in the private investment rate.

In conclusion, according to the results of these first econometric estimates, the long-term elasticity of the private investment rate with respect to the lending interest rate in the current period is positive ($\beta_{LT}>0$ and $\beta_{LTd}>0$) and higher than or equal to the absolute value of the long-term elasticity of the investment rate with respect to the lending interest rate of the past period.

b. Short run interest rate elasticity of private investment

Concerning the short-term static model, Hausman and Breusch and Pagan tests show that the individual random effects model is better suited for its estimation (Table 3). The random effects model with instrumental variables is used to reduce the estimation biases related to the problem of endogeneity of the lagged variable of the investment rate in the short-term static model. Wald's Chi2 statistic shows that the model is globally significant at the 1% level. Also, the restoring force at equilibrium (the coefficient of the lagged value of the investment rate) is negative (-0.2336) and significant at the 5% threshold and the interest rate elasticity of the private investment rate is positive (+0.2460) and significant at the 1% level.

Table 3: Results of estimates of short run models

Dependent variable: D.Log((GFCF private) /GDP) at t				
	Static panel model with random effects of the short run		**System Dynamic panel of short run**	
Explicatives variables	Coefficient	P>| t |	Coefficient	P>| t |
D.Log (Lending interest rate) at t	0.2460	0.000	0.2329	0.000
D.Log((GFCF private) /GDP) at t-1	//////	//////	0.1135	0.387
Log ((GFCF private)/GDP) at t-1	-0.2336	0.013	-0.1966	0.008
Constant	0.7087	0.013	0.5701	0.008
Hausman test	*Prob>Chi2*	*0.8455*		
LMM B & P test	*Prob>chi2*	*1.0000*		
Model quality statistics	*Prob>Wald chi2*	*0.000*	*Nb. Obs = 30*	*Prob>Wald chi2= 0.000*
	Nb. Obs.	*30*	***Differentiated equation*** ***GMM-Type***: L(2/.).D.Log(GFCF private/GDP) at t ***Standard***: DLog(GFCF private/GDP) at t-1, D2.Log(Interest rate) at t, **Level equation** ***GMM-Type:*** D2.Log(GFCF private/GDP) at t-1 ***Standard***: constant	
Instrumental variables	**Instrumented**: Log((GFCFprivate)/GDP) at t-1 **Instruments**: D.Log(interest rate), Log((GFCFprivate/GDP) at t-2			

Source: Based on WDI 2021 data, World Bank

About the short-term dynamic model, the results of the estimation show that the restoring force is also negative (-0.1966) and significant at the 1% threshold and the interest rate elasticity of private investment rate is positive (+0.2329) and significant at the 1%. However, the coefficient of the lagged variable of the change in the investment rate is not significantly different from 0 at the 10% threshold. As a result, the annual change in the investment rate would not have an autoregressive dynamic character. The results of the short-term static model would then be more relevant.

Thus, according to the results of these second econometric estimations, the coefficients β_{CT} and β_{CTd} are both positive and significantly different from 0, and the coefficients δ_1 and δ_{1d} of the restoring force are both negative and significantly different from 0.

In summary, over the period 1996-2012 in the United States and Japan, the interest rate elasticities of private investment of short and long term were positive and significantly different from 0 and the coefficient of the restoring force at equilibrium was negative and significantly different from 0 in both the dynamic and static models. This confirms the hypothesis that the fall in the lending interest rate observed in the USA and Japan between 1996 and 2012 had a negative effect on private investment.

6. Conclusion

Following more general reflections on the investment decision of companies within the framework of the General Theory of the Firm which showed that the relationship between investment and the interest rate is not monotonous, this paper set the purpose (i) to carry out more precise theoretical investigations to apprehend different conditions under which the said relationship is increasing or decreasing and (ii) to carry out econometric tests to validate or invalidate some of these detailed theoretical results. Thus, this work has made it possible to record important theoretical advances on the relationship between investment and the interest rate.

First, the expression of the interest rate elasticity of investment has been determined for behavior functions of the firm's stakeholders that can be represented in the form of Cobb-Douglas functions. From this expression, it emerges that the elasticity of investment with respect to the lending interest rate is a function of several economic variables: (i) the investment level, (ii) interest rates on the capital markets,

(iii) the expected retained earnings, (iv) the respective capital stocks financed by equity and borrowings, (v) the supplies of equity and loanable funds, as well as ((vi) corporate income tax rate.

Second, the study of the sign of the limit of the interest rate elasticity of investment when the ratio of the funds supply to the stock of capital relative to each capital market tends to zero showed that this elasticity can be positive, negative or equal to 0 in the event of accentuated rationing of the demand for corporate financing. Indeed, when the demand for corporate financing is heavily rationed, (i) the lending interest rate elasticity of investment is equal to 0 if the supply of loanable funds is not sensitive to changes in the lending interest rate; (ii) it is positive if (a) firms' aversion to paying interest is high, (b) the collateral system is relatively efficient, and (c) the supply of credit is relatively elastic with respect to the interest rate ; (iii) it is negative if the effectiveness of the guarantee system is quite limited and the supply of credit is not very sensitive to the interest rate.

Third, the study of the sign of the limit of the interest rate elasticity of investment when the lending interest rate tends to zero has shown that this elasticity can be positive or negative according to two different situations. First, when the lending interest rate is relatively low, the interest rate elasticity of investment is negative if the expected return rate on equity is less than the ratio of retained earnings in relation to the stock of capital so that their ratio is lower than the share of debts in the total capital of companies.

On the other hand, when the lending interest rate is relatively low, this elasticity is positive if: (i) the expected return rate on the equity market is higher than the ratio of the expected retained earnings in relation to the capital stock, (ii) the profitability of capital is higher than 0, (iii) the supply of equity is relatively elastic with respect to the expected net return rate on equity and (iv) the supply of equity is very high compared to the expected dividends. This means that the interest rate elasticity of investment is positive in periods of strong speculation on the stocks markets associated with low levels of lending interest rates.

This last theoretical result was confirmed by empirical tests on the United States and Japan during the period 1996-2012, a period of heavy speculation on the stocks markets associated with sharp declines in lending interest rates. This empirical evidence has shown that the long run and short run elasticities of investment with respect to the lending interest rate were positive in the United States and Japan during this period of strong speculation on stocks market. Thus, the decrease in lending interest rates in the United States and Japan between 1996 and 2012 had a negative impact on investment.

Thus, a standardized use of the interest rate as a tool for boosting private investment regardless of economic and financial conditions is inadequate and risky.

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